

$$I = \iint_D f(x, y) dx dy$$

$$x = \varphi(u, v)$$

$$y = \psi(u, v)$$

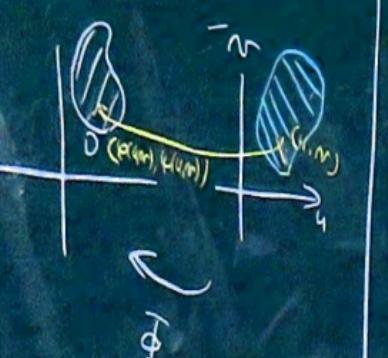
$$t = t_1,$$

$$\Phi: (u, v) \mapsto (\varphi(u, v), \psi(u, v))$$

C₁-級

平面領域 D' における $\Phi[D'] = D$

$t = t_1 = 3, \dots$



$$I = \iint_{D'} f(\varphi(u, v), \psi(u, v)) |J| du dv$$

$\Rightarrow I = \iint_{D'} f(\varphi(u, v), \psi(u, v)) |J| du dv$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \underbrace{\frac{\partial \psi}{\partial u} \frac{\partial \varphi}{\partial v}}_{\text{Jacobi determinant}} - \underbrace{\frac{\partial \psi}{\partial v} \frac{\partial \varphi}{\partial u}}$$

これは極座標変換

$$x = r \cos \theta$$

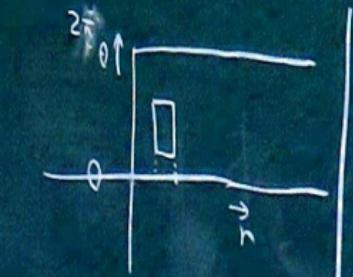
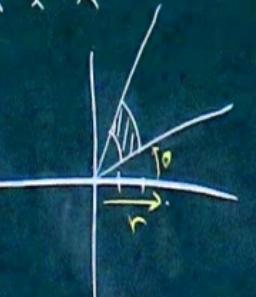
$$y = r \sin \theta$$

となる

$$J = r$$

$$r \geq 0$$

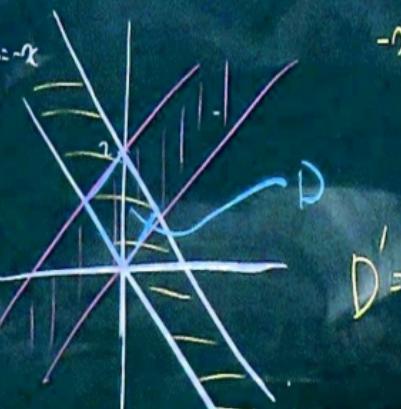
$$|J| = r$$



$$\begin{aligned} I &= \iint_D \sqrt{R^2 - (x^2 + y^2)} dx dy \\ &= \int_0^{2\pi} \int_0^R \sqrt{R^2 - r^2} r dr d\theta \quad D' = [0, R] \times [0, 2\pi] \end{aligned}$$

$$\begin{aligned} & \left(-\frac{1}{8} \cdot \frac{8}{3} (R^2 - r^2)^{\frac{3}{2}} \right)' = \sqrt{R^2 - r^2} \quad \text{r で}, \\ & I = \int_0^{2\pi} \left[-\frac{1}{3} (R^2 - r^2)^{\frac{3}{2}} \right] R \, d\theta \\ & = \int_0^{2\pi} -\frac{1}{3} (R^3) \, d\theta \\ & = \frac{1}{3} R^3 [\theta]_0^{2\pi} = \frac{2}{3} \pi R^3 \end{aligned}$$

(1) $D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x+y \leq 2, -2 \leq x-y \leq 0\}$



$$D' = \{(u, v) \in \mathbb{R}^2 \mid 0 \leq u \leq 2, -2 \leq v \leq 2\}$$

$$I = \iint_D (x-y) e^{\frac{x+y}{2}} \, dx \, dy$$

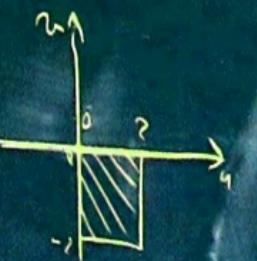
ここで $u = x+y, v = x-y$ とおく

$$\text{つまり, } x = \frac{u+v}{2}, y = \frac{u-v}{2} \text{ とおく.}$$

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

$$|J| = \frac{1}{2}$$

$$I = \frac{1}{2} \int_{-2}^0 \int_0^2 u v e^u \, du \, dv = \int_{-2}^0 v \left[e^u \right]_0^2 \, dv$$



$$= \frac{1}{2}(e^2 - 1) \int_{-2}^0 v \, dv$$

$$= \frac{1}{2}(e^2 - 1) \left[\frac{1}{2}v^2 \right]_{-2}^0$$

$$= \frac{1}{2}(e^2 - 1) \cdot \left(-\frac{1}{2} \cdot 8^2 \right)$$

$$= 1 - e^2$$

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$$D = \{(x, y) \mid x^2 + y^2 \leq R^2\}$$

$$D' = \{(r, \theta) \mid 0 \leq r \leq R, 0 \leq \theta \leq 2\pi\}$$

$I = \int_D \left(H - \frac{H}{R}\sqrt{x^2 + y^2}\right) dx dy$

$= \int_{D'} \left(H - \frac{H}{R}r\right) r dr d\theta$

$$\begin{aligned} &= \int_0^{2\pi} \int_0^R \left(Hr - \frac{H}{R}r^2\right) dr d\theta \\ &= \int_0^{2\pi} \left[\frac{1}{2}Hr^2 - \frac{1}{3}\frac{H}{R}r^3 \right]_0^R d\theta \\ &= \int_0^{2\pi} \left(\frac{1}{2}HR^2 - \frac{1}{3}\frac{H}{R}R^3 \right) d\theta \\ &\quad " \frac{1}{6}HR^2 \end{aligned}$$

$$= \left[\frac{1}{6}HR^3 \theta \right]_0^{2\pi}$$

$$= \frac{1}{3}HR^2 \cdot 2\pi$$

$$= \frac{1}{3}\pi R^2 H$$

$$\begin{aligned} &= \frac{1}{2}(e^2 - 1) \int_0^R r dr \\ &= \frac{1}{2}(e^2 - 1) \left[\frac{1}{2}r^2 \right]_0^R \\ &= \frac{1}{8}(e^2 - 1) \cdot \left(-\frac{1}{8}\pi\right) \\ &= 1 - e^2 \end{aligned}$$