Problem

\[ \text{Theorem C.1 (Zermelo-Fraenkel)} \]

(a) AC

(b) Well-ordering Theorem (Well-ordering set)

(i.e. every set has a well-ordering)

A pair \( (X, R) \) with \( R \) an irreflexive (i.e. \( a \neq a \) for all \( a \)) antisymmetric (i.e. \( a R b \Rightarrow b R a \) for all \( a, b \) in \( X \)) partial order is called a "partial ordering".

A partial ordering \( (X, R) \) is a linear ordering if for any \( Y \subseteq X \), \( Y \) contains the minimal element of \( Y \).

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where \( X = X \cup \mathbb{R} \)
\( R' = R \cup (X \times \mathbb{R}) \).

The \( \langle X, R' \rangle \) is a linear order,
(well-ordering, etc.) when
\[ X^* = U \{ (X, R) \in \mathcal{P}(X) \times \mathcal{P}(R) \mid (X, R) \in \mathcal{P}(X) \times \mathcal{P}(R) \} \]
\[ R^* = U \{ (X, R) \in \mathcal{P}(X) \times \mathcal{P}(R) \mid (X, R) \in \mathcal{P}(X) \times \mathcal{P}(R) \} \]

And \( \langle X, R' \rangle \) is an end-orientation of \( \langle X, R \rangle \).

(3) Suppose that \( \mathbb{N} \) is a set consisting of linear orderings
(well-ordering, etc.) p.t. for any \( \langle X, R \rangle \), \( X \neq \mathbb{N} \).
\( \mathbb{N} \) is a linear ordering of \( X \).
\( \mathbb{N} \) is an end-orientation of \( X \).

For a partial ordering \( \langle P, R \rangle \) \( \subseteq \mathbb{P} \)
\( \mathbb{R} \) is a chain if \( \langle C, R \rangle \) \( \subseteq \mathbb{R} \) is a linear ordering.

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For any set $X$

$$\emptyset \subseteq P(X)$$

If $\emptyset \in P(X)$ then $\forall x \in X \forall y \in x \Rightarrow y \in P(X)$

Proof:

$X = \{\emptyset\} \implies \emptyset \subseteq X$ for all $x \in X$

Let $x \in X$ and $y \in x$.

Then $\emptyset \subseteq x$.

Let $x \in X$ and $y \in x$.

Since $\emptyset \subseteq x$, then $y \in \emptyset$.

Therefore, $\emptyset \subseteq P(x)$.

Hence, $y \in P(x)$.

Thus, $y \in P(X)$.

Q.E.D.