Note that \( \{ \Phi_n \} \) converges to \( \Phi \) in the topology of \( \| \cdot \| \).

**Lemma 21.1:** \( A \subseteq \text{l}(\omega) \) is open iff for any \( \Phi A \) then \( \text{new} \) \( n \).

\[
[\Phi n] \subseteq A \iff [\Phi n] = \{ \Phi n \}
\]

**Proof:** Suppose \( A \) is open. By \( 21.0 \) there is a \( n \)-free formula \( \psi \) in \( \text{new} m, \text{new} n \) such as \( \Phi n \in \{ \Phi n \} \).

Suppose \( A \subseteq \text{l}(\omega) \) is open.

\[
\text{Let } \psi n = \Phi n \in \{ \Phi n \} \text{ new } m, \text{new} n \text{.}
\]

Let \( \psi n = \Phi n \in \{ \Phi n \} \text{ new } m, \text{new} n \text{.}

\[
\text{Suppose } \psi n \text{ is } n \text{-free and } \psi \text{ is } m \text{-free.}
\]

\[
A = \bigcup [\Phi n] \text{ if } \psi n \text{ new } m, \text{new} n \text{.}
\]
Lemma 2.1.2. (a) Suppose \( B = \{ f \in M(W) : A^2 = f(\mathbf{v}, \mathbf{a}) \} \) for an \( \mathfrak{g} \)-module \( \mathbf{v} \) and \( \mathbf{a} \).

Then \( \{ f \in M(W) : A^2 = f(\mathbf{v}, \mathbf{a}) \} = p(B) \).

(2) If \( B = \{ f \in M(W) : A^2 = f(\mathbf{v}, \mathbf{a}) \} \) is a subalgebra of \( \mathfrak{g} \), then \( \{ f \in M(W) : A^2 = f(\mathbf{v}, \mathbf{a}) \} = \{ \sum \mathbf{v} : f(\mathbf{v}, \mathbf{a}) \} \) is a subalgebra of \( \mathfrak{g} \).

Lemma 2.1.6. \( \mathfrak{g}^2 + A(\mathfrak{g}) \) is an ideal.

Then \( \{ f \in M(W) : A^2 = f(\mathbf{v}, \mathbf{a}) \} \) is a subalgebra of \( \mathfrak{g} \).

Proof: \( \mathfrak{g}^2 + A(\mathfrak{g}) \) is an ideal.

Then \( \{ f \in M(W) : A^2 = f(\mathbf{v}, \mathbf{a}) \} \) is a subalgebra of \( \mathfrak{g} \).

Note that \( \mathfrak{g} \) is a subalgebra of \( \mathfrak{g}^2 + A(\mathfrak{g}) \).
Consider the type formula $\Psi$:

$$\exists x \left( \phi(x) \land \forall z, z \in x \rightarrow \exists w, z \in \text{trail}(w, E_n) \right)$$

Then by induction on $n$:

- $\phi(x)$ is $\Sigma^1_n$ (it $A \in \Sigma^1_n$ is determined in $\langle H(n), \in \rangle$)
- By straightforward induction on $n$!