Lemma 10.1 (the winning detail in the proof of Theorem B3)

For any top. open \((X, \mathcal{O})\) and \(w_x\), if \(w(x) \leq x\), if \(\beta \in \text{open base of } (X, \mathcal{O})\), then \(\beta \in [\beta]\) is.

Let \(\beta = \{\beta_{n, p} : \beta_n, \beta_p \in \beta\} \in \beta\)

\(\beta_{n, p} \leq x \longmapsto \beta_p \in \beta\)

Corollary: \(B_p\) is open base of \(X\)

Claim \(B_p\) is open base of \(X\)

Suppose \(x \in X\) of \(\mathcal{O}\) of \(x\).

Let \(B_{x, 0}\) be an open base of \(x\).

Let \(B_{x, 0} \in \mathcal{O}\) be.

\(\beta_{x, 0} \in \mathcal{O}\) otherwise.

Than by \(\phi\) we have \(B_p \in \mathcal{B}\) but \(w(x) \in \mathcal{B}\).
Let $H \in \text{Th}(X)$ be with $(X, a, x) \in H$.
Since $d(x) \leq u(x)$ and $d(x) \leq u(x)$ for
some $x$ in $X$.
It is enough to show $d(x) \leq u(x)$. This follows from:
Claim: $X \setminus H$ is dense in $X$.
Proof: Suppose not. Then $X \setminus (X \setminus H) \neq \emptyset$.
Let $a \in X \setminus H$.
Let $U = \{0\} \cup \{U' \cup M \cap H, \emptyset \} \cup U \setminus X$.
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Suppose $U$ is a open covering of $X$.
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Let $M \cap H$ be with $(X, a, x) \in H$.
Since $d(x) \leq u(x)$ and $d(x) \leq u(x)$ for
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By the zero's assumption of $X$ there are
$U, V \in \mathcal{U}$ such that $U \cap V = \emptyset$.
Let $a \in X$.
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Let $M \cap H$ be with $(X, a, x) \in H$.
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Claim: $X \setminus H$ is dense in $X$.
Proof: Suppose not. Then $X \setminus (X \setminus H) \neq \emptyset$.
Let $a \in X \setminus H$.
Let $U = \{0\} \cup \{U' \cup M \cap H, \emptyset \} \cup U \setminus X$.
Let $U = \{0\} \cup \{U' \cup M \cap H, \emptyset \} \cup U \setminus X$.
Lemma 2.4 Suppose that $\mathcal{X}(x,y)$ is a locally compact Hausdorff space. Let $\mathcal{N}(x,y)$ be a neighborhood basis of $x$ with $\mathcal{N}(x,y) \subseteq \mathcal{N}(x,z) \subseteq \mathcal{N}(y,z)$ for all $x, y, z \in X$. Then $\mathcal{N}(x,y)$ is an open cover of $X$. To prove this, let $U = \{x \mid x \in \mathcal{N}(x,y) \} \subseteq \mathcal{N}(y,y)$ be an open cover of $X$. For each $z \in \mathcal{N}(x,y)$, let $O_z \subseteq \mathcal{N}(y,z)$ be a neighborhood of $z$ in $\mathcal{N}(y,z)$. Define $\mathcal{N}(x,y) = \{O_z \mid z \in \mathcal{N}(x,y) \}$.

Proof: If $x = \mathcal{N}(x,y)$, then $x$ is the union of the sets in $\mathcal{N}(x,y)$, i.e., $x = \bigcup_{z \in \mathcal{N}(x,y)} O_z = \mathcal{N}(x,y)$.

Assume $x \in X \setminus \mathcal{N}(x,y)$. Suppose towards a contradiction that each $x \in X \setminus \mathcal{N}(x,y)$ has a basis $\mathcal{B}(x)$ in $\mathcal{N}(x,y)$.

A contradiction is the proof of Thm 9.5!