Corollary 10.4: Suppose \( X \) is a Hausdorff space. If \( X \) is completely Hausdorff, then \( X \) is normal.

Proof: Let \( \langle M, \tau \rangle \in M(\mathcal{H}) \) be a completely Hausdorff space. By Lemma 10.3, \( \langle M, \tau \rangle \) is a normal space. By Lemma 9.3, it follows that \( \langle X, \tau \rangle \) is a normal space.

Corollary 9.3: \( X \) is a countably compact space if and only if \( X \) is a locally compact space.

Proof: \( X \) is countably compact if and only if every countable open cover of \( X \) has a finite subcover. This equivalence is known as the Aleph-null property.

Corollary 10.4: Suppose \( X \times Y \) is a Hausdorff space. If \( X \) is completely Hausdorff, then \( Y \) is normal.

Proof: Let \( \langle M, \tau \rangle \in M(\mathcal{H}) \) be a completely Hausdorff space. By Lemma 10.3, \( \langle M, \tau \rangle \) is a normal space. By Lemma 9.3, it follows that \( \langle Y, \tau \rangle \) is a normal space.

Theorem 10.4: Suppose \( X \times Y \) is a Hausdorff space. If \( X \) is completely Hausdorff, then \( X \times Y \) is normal.

Proof: Let \( \langle M, \tau \rangle \in M(\mathcal{H}) \) be a completely Hausdorff space. By Lemma 10.3, \( \langle M, \tau \rangle \) is a normal space. By Lemma 9.3, it follows that \( \langle X \times Y, \tau \rangle \) is a normal space.
The M1.3 (Axiom of Dependent Choice
Theorem)

If X is a non-empty set, then there exists a sequence (\langle a_n \rangle) of elements of X such that for all n, a_n \neq a_{n+1}.

Proof of Con M.4 from The M.3

Suppose that X is regular and countably compact (Heine-Borel). Then every Y \subseteq X is countably compact.

Then all Y \subseteq X are with a point-countable base. Hence by The M.3 it follows that X is countably compact.

\[ \text{Theorem 8.3 (Continuous Havana)} \]

For a topological space X, let B be a family of subsets of X. If B is open, then B is countably subcompact.

A metric space X has a locally countable base if and only if each base is countable.

The M.1.3

\[ \text{Assume that X is a zero-dimensional space.} \]

1. \text{X is countably compact.}
2. \text{X is not countably compact.}

Let A be an infinite set of points in X. Consider the set of all finite subsets of A.

\[ \text{Lemma 10.5} \]

For any regular \( \sigma \)-space X, and \( X \subseteq M \), then \( X \subseteq M \).

Proof: The proof is as follows... (Details follow...)
Let $(M, \mathcal{A}, \mathcal{N})$ be a topological space where $\mathcal{A}$ is the family of all open subsets of $\mathcal{N}$ containing $\mathcal{N}$.

By (2) and (3), we have $U(x) \supseteq x$.

By (4), $x \in \mathcal{N}$, and hence by (5), $\mathcal{N} \cap \mathcal{A} \neq \emptyset$.

By (6), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By Lemma 10.3, $\mathcal{N}$ is an open base for $\mathcal{A}$.

By Lemma 10.3, $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (7), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (8), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (9), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (10), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (11), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (12), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (13), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (14), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (15), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (16), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (17), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (18), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (19), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (20), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (21), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (22), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (23), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (24), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (25), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (26), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (27), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (28), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (29), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (30), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (31), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (32), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (33), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (34), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (35), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (36), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (37), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (38), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (39), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (40), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (41), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (42), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (43), $\mathcal{N}$ is an open base for $\mathcal{A}$.

By (44), $\mathcal{N}$ is an open base for $\mathcal{A}$.
On the other hand, we have

\[(\exists n) nN = (\exists m) [m \leq n] \Rightarrow \exists M \subseteq N\]

\[M \cap (\exists p \leq n) = \emptyset \implies M \subseteq N\]

Hence by \(\exists n\) and Lemma 10.4

\[(\exists m) \text{ is an open box of } (x^\exists M)(\exists n) = x^\exists M\]

A contradiction. \(\exists\)