微分積分学II　レポート No.2 解答

1. (1) $\displaystyle\int_1^2 (4x^5-3x^3+2x^2+7x-8)dx = \left[\frac{2}{3}x^6-\frac{3}{4}x^4+\frac{2}{3}x^3+\frac{7}{2}x^2-8x\right]_1^2 = \frac{455}{12}.$

   (2) $\displaystyle\int_2^1 \left(\sqrt[3]{x^2}+\frac{1}{\sqrt{x}}\right)dx = \left[\frac{3}{5}x^{\frac{5}{3}}+2x^{\frac{1}{2}}\right]_2^1 = \frac{13}{5}-\frac{6}{5}\sqrt[3]{4}-2\sqrt{2}.$

   (3) $\displaystyle\int_0^\pi \sin\left(\frac{4}{3}x\right)dx = \left[-\frac{3}{4}\cos\left(\frac{4}{3}x\right)\right]_0^\pi = -\frac{3}{4}\cos\left(\frac{4}{3}\pi\right)+\frac{3}{4}\cos(0) = \frac{3}{8}+\frac{3}{4} = \frac{9}{8}.$

2. (1) はじめに，ロピタルの定理より $\displaystyle\lim_{\varepsilon\to+0}\varepsilon\log\varepsilon = \lim_{\varepsilon\to+0}\frac{\log\varepsilon}{\frac{1}{\varepsilon}} = \lim_{\varepsilon\to+0}\frac{\frac{1}{\varepsilon}}{-\frac{1}{\varepsilon^2}} = \lim_{\varepsilon\to+0}(-\varepsilon) = 0.$

   $\displaystyle\int_0^1 \log x dx = \lim_{\varepsilon\to+0}\int_\varepsilon^1 \log x dx = \lim_{\varepsilon\to+0}[x\log x - x]_\varepsilon^1 = \lim_{\varepsilon\to+0}(-1-(\varepsilon\log\varepsilon-\varepsilon)) = -1.$

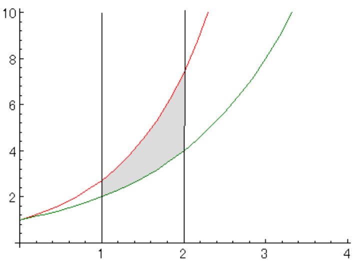
   (2) $\displaystyle\int_{-1}^1 \frac{1}{\sqrt{1-x^2}}dx = \lim_{\substack{\varepsilon'\to1-0\\ \varepsilon\to-1+0}}\int_\varepsilon^{\varepsilon'}\frac{1}{\sqrt{1-x^2}}dx = \lim_{\substack{\varepsilon'\to1-0\\ \varepsilon\to-1+0}}\left[\sin^{-1}x\right]_\varepsilon^{\varepsilon'}$

   $= \sin^{-1}1-\sin^{-1}(-1) = \pi.$

3.
   - $\lambda>1$ のとき，$\displaystyle\int_0^1\frac{1}{x^\lambda}dx = \lim_{\varepsilon\to+0}\int_\varepsilon^1\frac{1}{x^\lambda}dx = \lim_{\varepsilon\to+0}\left[\frac{1}{1-\lambda}x^{1-\lambda}\right]_\varepsilon^1$

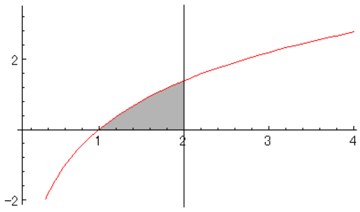
     $\displaystyle= \lim_{\varepsilon\to+0}\frac{1-\varepsilon^{1-\lambda}}{1-\lambda} = \infty.$
   - $\lambda=1$ のとき，$\displaystyle\int_0^1\frac{1}{x^\lambda}dx = \lim_{\varepsilon\to+0}\int_\varepsilon^1\frac{1}{x^\lambda}dx = \lim_{\varepsilon\to+0}(-\log\varepsilon) = \infty.$
   - $\lambda<1$ のとき，$\displaystyle\int_0^1\frac{1}{x^\lambda}dx = \lim_{\varepsilon\to+0}\int_\varepsilon^1\frac{1}{x^\lambda}dx \lim_{\varepsilon\to+0}\frac{1-\varepsilon^{1-\lambda}}{1-\lambda} = \frac{1}{1-\lambda}.$

4. (a)



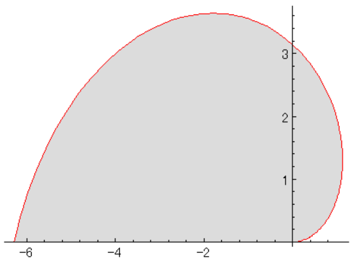
$$\int_1^2 (e^x-2^x)\,dx = \left[e^x-\frac{2^x}{\log 2}\right]_1^2 = e^2-e-\frac{2}{\log 2}.$$

(b)



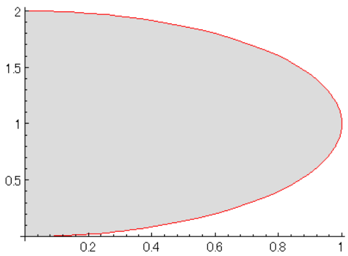
$$\int_1^2 (2\log x)\,dx = \int_1^2 (2(x)'\log x)\,dx = 2\,[x\log x]_1^2 - \int_1^2 1dx$$
$$= 2(2\log 2 - 0) - [x]_1^2 = 4\log 2 - 2.$$

5. (a)



$$\frac{1}{2}\int_0^{\pi} (2\theta)^2 d\theta = \frac{1}{2}\int_0^{\pi} 4\theta^2 d\theta = \frac{1}{2}\left[\frac{4}{3}\theta^3\right]_0^{\pi} = \frac{2}{3}\pi^3.$$

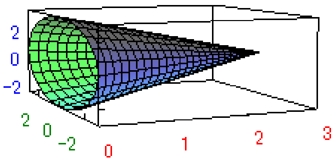
(b)



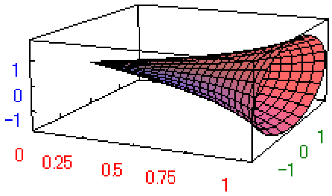
$$\frac{1}{2}\int_0^{\frac{\pi}{2}} (2\sin\theta)^2 d\theta = 2\int_0^{\frac{\pi}{2}} \sin^2\theta d\theta = 2\int_0^{\frac{\pi}{2}} \frac{1-\cos 2\theta}{2} d\theta$$
$$= 2\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{2}} = 2(\frac{\pi}{4} - 0) = \frac{\pi}{2}.$$

6. (a)

$$\int_0^3 \pi(3-x)^2dx = \left[-\frac{\pi}{3}(3-x)^3\right]_0^3 = 0 + \frac{\pi}{3}3^3 = 9\pi.$$



(b)



$$\int_0^{\frac{\pi}{3}} \pi(\tan x)^2dx = \pi\int_0^{\frac{\pi}{3}} \frac{\sin^2 x}{\cos^2 x}dx = \pi\int_0^{\frac{\pi}{3}} \frac{1}{\cos^2 x} - 1dx$$
$$= \pi\left[\tan x - x\right]_0^{\frac{\pi}{3}} = \pi(\sqrt{3} - \frac{\pi}{3}).$$