Linear Algebra 2　Exercises

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1. Calculate the inverse of the following 3×3-matrix:

$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & -1 \\ -5 & 5 & 2 \end{bmatrix}$$

2. Prove the following assertion by checking the definition of the inverse of a square matirix:

For any 2×2-matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, A is invertible if and only if $det(A) = ad - bc \neq 0$, and the inverse of A is

$$A^{-1} = \frac{1}{det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

3. Prove that the following hold for any $n \in \mathbb{N}$:

(1) If $\tau \in S_n$ then $\tau^{-1} \in S_n$.

(2) For τ, $\sigma \in S_n$, we have $\tau\sigma \in S_n$ where $\tau\sigma$ denotes the compotition of functions σ and τ.

(3) $(\pi\tau)\sigma = \pi(\tau\sigma)$ holds for all π, τ, $\sigma \in S_n$.

(4) S_n has $n!$ elements.

4. Let $\tau : \{1, ..., 5\} \to \{1, ..., 5\}$ be a permutation of degree 5 defined by $\tau = (1\ 2)(3\ 4)(2\ 5)$. Find $\tau(2)$, $\tau(4)$, $\tau(5)$. What is τ^{-1}?

5. (1) Enumerate all elements of S_3.

Use the notation: For $\tau : \{1, ..., n\} \to \{1, ..., n\}$ with $\tau(1) = t_1$, $\tau(2) = t_2$,..., $\tau(n) = t_n$, τ is denoted by

$$\tau = \begin{pmatrix} 1 & 2 & \cdots & n \\ t_1 & t_2 & \cdots & t_n \end{pmatrix}.$$

(2) Represent all elements of S_3 as products of transpositions.