Linear Algebra I Exercises II

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This list of exercises (and its possible further update) is downloadable as:

`http://kurt.scitec.kobe-u.ac.jp/~fuchino/kobe/lin-alg1-ss14-ex2.pdf`

Solutions of these exercises are to be submitted at the supplementary lecture held on July 24.

Some other materials connected to to the lecuture might be found at:

`http://kurt.scitec.kobe-u.ac.jp/~fuchino/kobe/index.html`

A lecture note of the course will be also linked to this page in the course of the semester.

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1. Find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix},$$

if it exists.

2. Use the solution of **1.** to obtain the solution of

$$\begin{cases} \quad\quad y + 2z = 1 \\ x \quad\quad + 3z = 2 \\ 4x - 3y + 8z = 3. \end{cases}$$

3. Show that, for $\mathbf{a}_1, \ldots, \mathbf{a}_n \in \mathbb{R}^n$, the $n \times n$-matrix $A = [\mathbf{a}_1 \cdots \mathbf{a}_n]$ is invertible if and only if $\mathbf{a}_1, \ldots, \mathbf{a}_n$ are linearly independent.

4. Show that $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ are not linearly independent.

5. Use **3.** and **4.** to decide if the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

is invertible.

6. Compute

$$\begin{vmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \end{vmatrix}.$$

7. Compute the following determinants:

(a) $\begin{vmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{vmatrix}$.

8. Use **7.**, (b) to calculate the volume of the parallelopiped spanned by the vectors $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} -12 \\ 3 \\ 12 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$.