Linear Algebra II – Exercise on Jan. 9 and its solution (2014/01/20)

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Diagonalize the matrix $A = \begin{bmatrix} 5 & -6 \\ 2 & -2 \end{bmatrix}$.

A possible Solution:

(Note that the solution is not unique since we can enumerate eigenvalues in different order and eigenvectors can be chosen differently.) The characteristic polynomial for the matrix is:

$\det|A - \varepsilon E| = \det \begin{vmatrix} 5-\varepsilon & -6 \\ 2 & -2-\varepsilon \end{vmatrix} = (\varepsilon - 2)(\varepsilon - 1)$.

Thus $\det|A - \varepsilon E| = 0 \iff \varepsilon = 2$ or $\varepsilon = 1$. That is, the eigenvalues of A are 1 and 2.

$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is one of the solutions of $Ax = x$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is one of the solutions of $Ax = 2x$. Hence, we obtain

$$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 5 & -6 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}.$$

Or, since $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$, we can write this as

$$\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 5 & -6 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}.$$