Linear Algebra II – Exercise 3 (2013/11/26)

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Lecture note and some other materials are linked at

`http://kurt.scitec.kobe-u.ac.jp/~fuchino/kobe/index.html`

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(1) Calculate the determinant of the following matrices:

(a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 5 & 7 \\ 1 & 3 & 9 & 27 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 9 & 6 & 7 \\ 0 & 6 & 1 & 4 \\ 0 & 9 & 1 & 0 \\ 1 & 4 & 4 & 0 \end{bmatrix}$

(2) Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Show that $\varphi_A \circ \varphi_B(\mathbf{e}_i^3) = \mathbf{e}_i^3$ holds for all $i = 1, 2, 3$.

(b) Conclude from (a) that $\varphi_A \circ \varphi_B = id_{\mathbb{R}^3}$.

(c) Use (b) to show $AB = E_3$.

(d) For any mappings $f : X \to Y$ and $g : Y \to X$, show that f is 1-1 and g is onto if we have $g \circ f = id_X$.

(e) Show that φ_B is 1-1 but not onto and φ_A is onto but not 1-1.

(f) Determine $\mathrm{Im}(\varphi_B)$ and $\mathrm{Ker}(\varphi_A)$.

(3) Let $\varphi_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ be the rotation counter-clockwise through the angle θ around the origin.

(a) Show that φ_θ is a linear mapping.

(b) Determine $M(\varphi_\theta)$.

(c) Show that φ_θ is a bijection.

(4) Answer which of the following mappings are linear. Explain why. Determine $M(\varphi_i)$ for the linear mappings φ_i among the following.

(a) $\varphi_1 : \mathbb{R}^2 \to \mathbb{R}^2; \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x+1 \\ y+1 \end{bmatrix}$ (b) $\varphi_2 : \mathbb{R}^2 \to \mathbb{R}^2; \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x+y \\ x-y \end{bmatrix}$

(c) $\varphi_3 : \mathbb{R}^2 \to \mathbb{R}^2; \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x\sin\theta \\ y\cos\theta \end{bmatrix}$ (d) $\varphi_4 : \mathbb{R}^2 \to \mathbb{R}^3; \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

(e) $\varphi_5 : \mathbb{R}^2 \to \mathbb{R}^3; \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} y \\ 0 \\ x \end{bmatrix}$ (f) $\varphi_6 : \mathbb{R}^2 \to \mathbb{R}^4; \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

(g) $\varphi_7 : \mathbb{R} \to \mathbb{R}^2; x \mapsto \begin{bmatrix} x \\ x^2 \end{bmatrix}$