SAMPLE

1   Let $A = \begin{bmatrix} 1 & 0 & -3 \\ -3 & 1 & 6 \\ 2 & -2 & -1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$.

(a)   Find $det\, A$

(b)   Find A^{-1}.

(c)   Solve the equation $A\mathbf{x} = \mathbf{b}$.

2   Let $A = \begin{bmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ where b represents a number.

(a)   Decide for which b the matrix A is invertible.

(b)   Find A^{-1} for such b that A is invertible.

(c)   Decide for which b the equation $A\mathbf{x} = \mathbf{0}$ has a non trivial solution.

3   Find the standard matrices of the following linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$:

(a)   Reflection through the x_2-axis.
(b)   Reflection through the origin.
(c)   Horizontal expansion by factor k (i.e. $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} kx \\ y \end{bmatrix}$)

4   Find the standard matrix of the linear transformation

$$\varphi : \mathbb{R}^2 \to \mathbb{R}^3; \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} 3x_1 + x_2 \\ 5x_1 + 7x_2 \\ x_1 + 3x_2 \end{bmatrix}.$$

5   Show that, for an $n \times n$ matrix A, 0 is one of its eigen values if and only if A is not invertible.

6   Suppose that λ_1 and λ_2 are two distinct eingen values of a matrix A and $\mathbf{u}$ and $\mathbf{v}$ are eigen vectors corresponding to them respectively. Show that $\mathbf{u}$ and $\mathbf{v}$ are linearly independent.