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数理論理学特論 3 Exercises for a report

2018 年 5 月 26 日

Participants of the course on mathematical logic are requested to submit a report on the following exercises. The grading will be done according to the report.

Solve (at least 1 and 2 but preferably all of) the following exercises and report on your solutions (with as much detailed explanations as possible).

Deadline: June 15, 2018 (an emvelope to put reports to submit will be set on the door of my office).

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1 Work out any of the applications of the method of elementary submodels treated in the course. Create a detailed explanations and fill all the gaps in the proof or find a more elegant alternative proof if possible.

2 Suppose that θ is a regular cardinal $> \aleph_1$ and $M \prec \mathcal{H}(\theta)$ is countable. Prove the following:

(a) $\omega \in M$, $\omega_1 \in M$.

(b) If $a_0,\ldots, a_{n-1} \in M$ then $\{a_0, \ldots, a_{n-1}\} \in M$.

(c) If $a \in M$ and a is countable then $a \subseteq M$.

(d) If $\langle a_\xi : \xi < \delta\rangle \in M$ then $\delta \in M$.

(e) $\omega_1 \cap M \in \omega_1$.

3

(a) Prove that, if $M \prec \mathcal{H}(\theta)$ is internally approachable (as defined in the 11th lecture), then M is internally cofinal (as defined in the 8th lecture).

(b) Prove the following generalization of Lemma 8.2:

Theorem. For any regular $\theta > \omega_1$ and $X \in [\mathcal{H}(\theta)]^{\leq\aleph_1}$, there is an internally approachable $M \prec \mathcal{H}(\theta)$ such that $X \subseteq M$.

4 Provide a detailed proof of Theorem 13.5.

5 $C \subseteq [\kappa]^{\aleph_0}$ is said to be club if (1) C is cofinal in $[\kappa]^{\aleph_0}$ with respect to $\subseteq$ and (2) for any $\subseteq$-increasing sequence $c_i \in C$, $i \in \omega$ $\bigcup_{i\in\omega} c_i \in C$. $S \subseteq [\kappa]^{\aleph_0}$ is stationary if $S \cap C \neq \emptyset$ for any club $C \subseteq [\kappa]^{\aleph_0}$.
Formulate and prove a Lemma corresponding to Lemma 6.8 for this notion of clubness and stationarity.