1. Lecture: Introduction and overview of the lectures.

An overview of the results which will be treated in the following lectures is given. In this first lecture, most of the results are mentioned without any proof or only with a very rough sketch of the proof. Detailed proofs of many of the results cited here will be given later at some appropriate places in the following lectures.

2. Lecture: The method of elementary submodels (I)

We introduce the basics of the method of elementary submodels of $\mathcal{M} = \langle \mathcal{H}(\theta), \in, \sqsubseteq, \cdots \rangle$. As an application, we give a proof of Δ-System Lemma using this method.

3. Lecture: The method of elementary submodels (II)

We introduce the notion of internally approachable models and internally unbounded (or internally cofinal; also called ω-bounding in old literature) models and we show some of the basic facts in connection with these notions. As the first example of substantial application of the method of elementary submodels to questions in topology, we prove the theorem of I. Juhász on reflection of uncountable weight of a topological space down to a subspace of cardinality $\aleph_1$.

4. Lecture: Dow's Reflection Theorem

We formulate Dow's Reflection Theorem on (non-)metrizability of countably compact topological spaces which was already mentioned in the first lecture. This theorem is one of the few "mathematical" theorems which was proved by the method of elementary submodels and whose only known natural proof is in terms of this method.
Some basic results in topology in preparation for the proof of Dow's Theorem are also proved in this lecture.

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5. Lecture: Proof of Dow's Reflection Theorem

We prove the Dow's Reflection Theorem building on the lemmas introduced in the 3. and 4. lectures.

The material in the 3., 4. and the present lectures are mostly included in the extended version of one of my articles downloadable as:

`http://kurt.scitec.kobe-u.ac.jp/~fuchino/papres/papers/balogh-x.pdf`

6. Lecture: Lecture: Stationary subsets of λ and $[\lambda]^{\aleph_0}$ (I)

We introduce the notion of closed unboundedness and stationarity of subsets of λ and $[\lambda]^{\aleph_0}$. We prove some of the basic facts about these notions, Fodor's Lemma and the Generalized Fodor's Lemma (Pressing Down Lemma), in particular.

Reflection Principles on stationarity of subsets of λ (OR) and $[\lambda]^{\aleph_0}$ (RP) for regular $\lambda > \aleph_1$ as well as Rado's Conjecture (RC) are introduced.

7. Lecture: Stationary subsets of λ and $[\lambda]^{\aleph_0}$ (II)

We prove the stationarity and co-stationarity of a certain type of subsets of $[\lambda]^{\aleph_0}$ which appear in the formulation of Fodor-type Reflection Principle introduced in the next lecture.

8. Lecture: Fodor-type Reflection Principle (FRP)

We introduce the Fodor-type Reflection Principle (FRP) and prove some basic facts about the principle. We show that each of RP and RC implies FRP (and it is apparent from definitions that FRP implies OR).

9. Lecture: Characterizations of FRP (I)

We begin the proof of a set-theoretic characterization of FRP which is used to prove the results in the 11. lecture.

10. Lecture: Characterizations of FRP (II)

We finish the proof of the set-theoretic characterization of FRP in the 9. lecture.

11. Lecture: FRP and topological Reflection Theorems

We show that some topological reflection theorems (notably the reflection of the (non-)metrizability of locally countably compact spaces, in extension of Dow's Reflection Theorem) are equivalent to FRP over ZFC.
We also show via. topological result obtained here that FRP implies the total failure of the square principle which implies that FRP is a large cardinal property and this suggests that some fairly large cardinal is needed to prove the consistency of the principle. In the 14. lecture we show that the consistency of FRP can be shown starting from a strongly compact cardinal.

12. Lecture: Summary of basics of forcing

We give a summary of some basic definitions and facts about the forcing. The knowledge of the forcing needed in the following lectures is well covered by the Chapter VII of:

◦ K. Kunen, Set Theory, an Introduction to Independence Proofs, North-Holland (1980).

This lecture is going to be based on the lecture note

`http://kurt.scitec.kobe-u.ac.jp/ fuchino/shizuoka/forcing2010.pdf`

of the lectures in a summer school I gave in 2007 (unfortunately written in Japanese). If I have enough time until this lecture, I will prepare a handout with the summary of the definitions of the connected notions and results of forcing needed in the next three lectures.

13. Lecture: Preservation of FRP by c.c.c. forcing

We show that FRP is preserved under the c.c.c. forcing extension. From this, it follows that FRP is strictly weaker than RP and RC, and that the topological reflection principle considered in the 11. lecture are consistent with arbitrarily large size of the continuum and/or Martin's Axiom. In contrast, it is known that each of RP and RC implies $2^{\aleph_0} \leq \aleph_2$ and RC is inconsistent with Martin's Axiom.

14. Lecture: Consistency proof of FRP.

We show that we can force FRP under the existence of a strongly compact cardinal. Almost the same proof also shows the consistency of RC.

For further reading in connection with the materials treated in the 9., 10., 11., 13./ and the present lectures:

∘ S. Fuchino, I. Juhász, L. Soukup, Z. Szentmiklóssy and Toshimichi Usuba, Fodor-type Reflection Principle and reflection of metrizability and meta-Lindelöfness, Topology and its Applications Vol.157, 8 (June 2010), 1415–1429.

`http://kurt.scitec.kobe-u.ac.jp/~fuchino/papers/ssmL-erice.pdf`

∘ S. Fuchino, H. Sakai and T. Usuba, More about Fodor-type Reflection Principle, preprint.

`http://kurt.scitec.kobe-u.ac.jp/ fuchino/papers/moreFRP.pdf`

15. Lecture: Further Results and Open problems.

As the last remarks, we mention other known results in connection with FRP and some (what I think very interesting) open questions. I shall also mention some further reference in connection with the reflection principles.