Reflection of non-metrizability

my first beamer slides
はじめてのびぃまぁ

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組合せ論的集合論と記述集合論
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京都大学数理解析研究所
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The results presented in this talks are mainly obtained in a joint research with:

István Juhász, Lajos Soukup,
Zoltán Szentmiklóssy and
薄葉 季路

- Fuchino Sakaé, Juhász István, Soukup Lajos, Szentmiklóssy Zoltán and Usuba Toshimichi, Fodor-type Reflection Principle, metrizability and meta-Lindelöfness, preprint.
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Reflection Principles

Axiom R is the principle asserting that the following $\mathrm{AR}([\kappa]^{\aleph_0})$ holds for all cardinals $\kappa \geq \aleph_2$:

$\mathrm{AR}([\kappa]^{\aleph_0})$: For any stationary $S \subseteq [\kappa]^{\aleph_0}$ and ω_1-club $T \subseteq [\kappa]^{\aleph_1}$, there is $I \in T$ s.t. $S \cap [I]^{\aleph_0}$ is stationary in $[I]^{\aleph_0}$.

For an uncountable set X, $T \subseteq [X]^{\aleph_1}$ is said to be ω_1-club if

- T is cofinal in $[X]^{\aleph_1}$ with respect to $\subseteq$; and
- T is closed with respect to the union of increasing chain of length ω_1.

Stationary Reflection (SR) of subsets of κ:

$\mathrm{SR}(\kappa)$: For any stationary $S \subseteq E^{\kappa}_{\omega} = \{\alpha < \kappa : \mathrm{cf}(\alpha) = \omega\}$ there is a $\alpha \in E^{\kappa}_{\omega_1}$ s.t. $S \cap \alpha$ is stationary in α.

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In some/many cases we can replace $\mathrm{RP}([\kappa]^{\aleph_0})$ by its weaker version $\mathrm{WRP}([\kappa]^{\aleph_0})$ obtained by dropping the condition $\mathrm{cf}(I) = \omega_1$.

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Some (well-known) facts about reflection principles

Assume that κ is regular and $\geq \aleph_2$.

- $\mathrm{AR}([\kappa]^{\aleph_0}) \Rightarrow \mathrm{RP}([\kappa]^{\aleph_0})$

  [$\{I \in [\kappa]^{\aleph_1} : \omega_1 \subseteq I,\ \mathrm{cf}(I) = \omega_1\}$ is ω_1-club.]

- $\mathrm{RP}([\kappa]^{\aleph_0}) \Rightarrow \mathrm{SR}(\kappa)$

  [For stationary $S \subseteq E^{\kappa}_{\omega}$, let

  $$\tilde{S} = \{a \in [\kappa]^{\aleph_0} : \mathrm{cf}(a) = \omega,\ \sup(a) \in S\}.$$

  Apply $\mathrm{RP}([\kappa]^{\aleph_0})$ on $\tilde{S}$.]

- $\mathrm{AR}([\omega_2]^{\aleph_0}) \Leftrightarrow \mathrm{RP}([\omega_2]^{\aleph_0})$
- It is open if $\mathrm{AR}([\kappa]^{\aleph_0})$ and $\mathrm{RP}([\kappa]^{\aleph_0})$ are equivalent for $\kappa > \omega_2$
- $\mathrm{SR}(\kappa) \not\Rightarrow \mathrm{RP}([\kappa]^{\aleph_0})$

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Fodor-type reflection principle

Assume that κ is regular and $\geq \aleph_2$

$$\mathrm{AR}([\kappa]^{\aleph_0}) \implies \mathrm{RP}([\kappa]^{\aleph_0}) \implies \mathrm{SR}(\kappa)$$

$$\mathrm{RP}([\kappa]^{\aleph_0}) \searrow \mathrm{FRP}(\kappa) \nearrow \mathrm{SR}(\kappa)$$

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Lemma 1

For a cardinal $\kappa \geq \aleph_2$, $\mathrm{FRP}(\kappa)$ is equivalent to the following $\mathrm{FRP}^\bullet(\kappa)$:

$\mathrm{FRP}^\bullet(\kappa)$: *For any stationary $S \subseteq E^\kappa_\omega$ and mapping $g : S \to [\kappa]^{\leq\aleph_0}$ there is a continuously increasing sequence $\langle I_\xi : \xi < \omega_1 \rangle$ of countable subsets of κ s.t.*

- $\langle \sup(I_\xi) : \xi < \omega_1 \rangle$ *is strictly increasing;*
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For any cardinal κ with $cf(\kappa) \geq \omega_1$, $\mathrm{RP}([\kappa]^{\aleph_0})$ implies $\mathrm{FRP}(\kappa)$.

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Applications of FRP (1/2)

The following theorems originally obtained under Axiom R can be proved under FRP:

Theorem 3 (W.Fleissner, 1986)

(Axiom R) *For a T_1 space X with point-countable base, if every subspace Y of X of cardinality $\leq \aleph_1$ is left separated (i.e. Y can be enumerated as $\{y_\alpha : \alpha < \lambda\}$ s.t. $\{y_\alpha : \alpha < \delta\}$ for all $\delta \leq \lambda$ is closed) then X is also left separated.* □

Theorem 4 (Q.Feng (and S.F.), 1994)

(Axiom R) *For a Boolean algebra B s.t. , for any $X \in [B]^{\aleph_1}$, there is a relatively complete subalgebra C of B of cardinality $\aleph_1$ with $X \subseteq C$, if there are cofinally many openly generated subalgebras C of B of cardinality $\aleph_1$, then B is openly generated.* □

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The following theorem also can be proved under FRP:

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The assertion of Theorem 5 follows from the following:

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Recall that a topological space X is meta-Lindelöf if every open cover has a point countable refinement.

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Preservation of FRP by c.c.c. forcings

Theorem 7 (S.F., I.Juhász, L.Soukup,Z.Szentmiklóssy and T.Usuba)

Suppose that FRP *holds. If* $\mathbb{P}$ *is a c.c.c. poset. Then we have* $\Vdash_{\mathbb{P}}$ " FRP ". □

Corollary 8

FRP *does not imply* RP.

Proof. RP implies $2^{\aleph_0} \leq \aleph_2$. By Lemma 7, FRP does not imply this. □ (Corollary 8)

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*The assertions of Fleissner's theorem (*Theorem 3*), Q.Feng(+S.F.)'s theorem (*Theorem 4*), Balogh's theorem (*Theorem 5*) etc. are consistent with arbitrary large size of continuum.* □

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