Topological characterizations of some set-theoretic principles

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Theorem 1 (S.F., Juhász, Soukup, Szentmiklóssy, Usuba [1])

Assume Fodor-type Reflection Principle (see the next slide). Then the following reflection theorem on metrizability holds:

> *For a locally countably compact topological space X, if all subspaces Y of X cardinality $\leq \aleph_1$ are metrizable then X itself is metrizable.*

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FRP is a strengthening of the Ordinal Reflection Principle (ORP):

For any regular cardinal $\kappa > \aleph_1$ and any stationary $S \subseteq E^\kappa_\omega$, there is an $\alpha < \kappa$ with $\mathrm{cf}(\alpha) = \omega_1$, s.t. $S \cap \alpha$ is stationary in α.

with the side condition which reminds of Fodor's Theorem:

Fodor's Theorem. For any regular cardinal κ and $f : \kappa \to \kappa$ s.t. $f(\alpha) < \alpha$ for all $\alpha < \kappa$, there is a $\xi^* < \kappa$ s.t. $f^{-1}{}''\{\xi^*\}$ is stationary in κ.

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The reflection theorem on metrizability of locally countably compact spaces in Theorem 1 is equivalent to FRP *over* ZFC.

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Generalization of Theorem 2

Theorem 2 can be obtained via the proof of the following result:

Theorem 3 (S.F. et al. [1]; and S.F. et al. [2])

FRP *is equivalent to the following assertion:*

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Further results (1/3)

Theorem 4 (S.F. [3]; and S.F. Sakai, Soukup and Usuba [2])

The following assertion is equivalent to FRP*:*

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The following assertion is equivalent to FRP*:*

For a countably tight space of local density $\aleph_1$, if all subspaces Y of X of cardinality $\leq \aleph_1$ are collectionwise Hausdorff then X itself is collectionwise Hausdorff.

- A space X is *of local density κ* if for very $p \in X$ there is $Y \in [X]^{\leq\kappa}$ s.t. $p \in int(\overline{Y})$.

- A space X is *collectionwise* Hausdorff if any closed discrete subset D of X can be simultaneously separated by disjoint open sets, i.e., if, for any closed and discrete $D \subseteq X$, there is a family $\mathcal{U}$ of pairwise disjoint open sets such that, for all $d \in D$, there is $U \in \mathcal{U}$ with $D \cap U = \{d\}$.

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- FRP is also equivalent to the reflection theorems in terms of:
  - countable coloring number of infinite graphs (S.F. et al.[2]); and
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Applications

- The topological (graph-theoretic, Boolean algebraic) reflection principles mentioned above are all equivalent to each other over ZFC.

- All these reflection principles imply Ordinal Reflection Principle.

Theorem 6 (S.F. et al. [1])

FRP *is preserved by c.c.c. generic extension.*

- Hence, all reflection principles above impose almost no restriction on the size of continuum. **cf.:** Under slightly stronger reflection principles, the continuum is $\leq \aleph_2$.

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FRP *implies Shelah's Strong Hypothesis.*

- *Shelah's Strong Hypothesis* (SSH) is the assertion equivalent to the following:

> For every uncountable cardinal κ of countable cofinality, we have $cf([\kappa]^{\aleph_0}, \subseteq) = \kappa^+$.

- By the characterization above of SSH, Singular Cardinal Hypothesis (SCH) follows from SSH.

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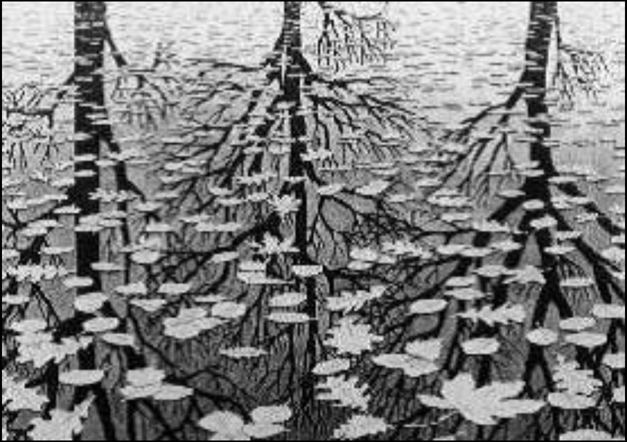
Thank you for your attention!



A part of: M.C. Escher, "Three Worlds" (1955)

These slides and their printer friendly version are linked to:

http://kurt.scitec.kobe-u.ac.jp/~fuchino/