Fodor-type Reflection Principle and its "mathematical" characterizations

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Co-authors and references

The most recent results in this talk are obtained in a joint research with:

Lajos Soukup (Budapest), Hiroshi Sakai (Kobe), and Toshimichi Usuba (Bonn)

Related Papers and Preprints

[1] S. Fuchino, I. Juhász, L. Soukup, Z. Szentmiklóssy and T. Usuba, *Fodor-type Reflection Principle and reflection of metrizability and meta-Lindelöfness*, to appear in Topology and Its Applications.

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Reflection Principles

$\mathrm{RP}([\lambda]^{\aleph_0})$: For any stationary $S \subseteq [\lambda]^{\aleph_0}$, there is an $I \in [\lambda]^{\aleph_1}$ s.t. $\omega_1 \subseteq I$, $cf(I) = \omega_1$ and $S \cap [I]^{\aleph_0}$ is stationary in $[I]^{\aleph_0}$.

$\mathrm{AR}([\lambda]^{\aleph_0})$: For any stationary $S \subseteq [\lambda]^{\aleph_0}$ and ω_1-club $\mathcal{T} \subseteq [\lambda]^{\aleph_1}$, there is $I \in \mathcal{T}$ s.t. $S \cap [I]^{\aleph_0}$ is stationary in $[I]^{\aleph_0}$.

Here, $\mathcal{T} \subseteq [X]^{\aleph_1}$ for an uncountable set X is said to be ω_1-club (or “tight and unbouded” in Fleissner’s terminology) if

- $\mathcal{T}$ is cofinal in $[X]^{\aleph_1}$ with respect to $\subseteq$ and
- for any increasing chain $\langle I_\alpha : \alpha < \omega_1 \rangle$ in $\mathcal{T}$ of length ω_1, we have $\bigcup_{\alpha<\omega_1} I_\alpha \in T$.

RP $:\Leftrightarrow$ $\mathrm{RP}([\lambda]^{\aleph_0})$ holds for all cardinals $\lambda \geq \aleph_2$.

Axiom R $:\Leftrightarrow$ $\mathrm{AR}([\lambda]^{\aleph_0})$ holds for all cardinals $\lambda \geq \aleph_2$.

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$\mathrm{MA}^+(\sigma\text{-closed}) \Rightarrow$ Axiom R $\Rightarrow \mathrm{RP}$

Set-theoretic consequences of RP

- (Todorcevic) $2^{\aleph_0} \leq \aleph_2$
- (Foreman, Magidor Shelah) Every poset preserving stationarity of subsets of ω_1 is semiproper. As consequences of this we have e.g.:
- I_{NS} is precipitous
- A strong form of Chang's conjecture
- ...

Mathematical consequences of Axiom R

- Fleissner's Theorem on left-separated spaces
- Fleissner's Theorem on coloring number of graphs
- A characterization of openly generated Bas (F., Qi Feng)
- Balogh's reflection theorem on metrizability
- Balogh's "Theorem 1.4", "Theorem 1.6"
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Fleissner's Theorem on left-separated spaces

Theorem 1 (W. Fleissner 1986)

Assume Axiom R. Suppose that X is a T_1-space with a point countable base. If X is not left-separated then there is a subspace Y of X of cardinality $\leq \aleph_1$ which is not left-separated.

- A topological space X is left-separated if there is a well-ordering $<$ of X s.t. every initial segment with respect to $<$ is a closed subset of X.

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Fleissner's Theorem on coloring number of graphs

Theorem 2 (W. Fleissner 1986)

Assume Axiom R. If a graph (V, E) has coloring number $\geq \aleph_1$ then there is an infinite subgraph of (V, E) of cardinality $\aleph_1$ with coloring number $\aleph_1$.

- For a graph (V, E) the coloring number of (V, E) is the minimal cardinal μ s.t.

  there is a well-ordering $\prec$ of V s.t. , for every $v \in V$, the set $\{u \in V \,:\, u \prec v,\ \{u, v\} \in E\}$ has cardinality $< \mu$.

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Balogh's reflection theorem on metrizability

Theorem 3 (Z. Balogh 2002)

Assume Axiom R. Suppose that X is locally countably compact. If X is not metrizable then there is a subspace Y of X of cardinality $\leq \aleph_1$ which is not metrizable.

- A topological space X is coutably compact if any countable open cover of X has a finite subcover.
- A topological space X is locally countably compact if any point of X has a neighborhood which is countably compact.

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Balogh's reflection theorem on metrizability

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Theorem 4 (Z. Balogh 2002 (Theorem 1.4))

Assume Axiom R. Suppose that X is locally Lindelöf, countably tight and s.t.

> *for every subspace Y of X of Lindelöf degree $\leq \aleph_1$ $\overline{Y}$ has Lindelöf degree $\leq \aleph_1$.*

If X is not paracompact then there is a clopen subspace Y of X of Lindelöf degree $\leq \aleph_1$ which is not paracompact.

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- X is countably tight if $x \in \overline{Y}$ for any $x \in X$ and $Y \subseteq X$ then there is a countable $Y' \subseteq Y$ s.t. $x \in \overline{Y'}$.
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Fodor-type reflection principle (FRP)

The 'mathematical' reflection theorems mentioned above are actually consequences of the following combinatorial principle which is much weaker than Axiom R (even weaker than RP):

$\mathrm{FRP}(\kappa)$: For any stationary $S \subseteq E^{\kappa}_{\omega}$ and mapping $g : S \to [\kappa]^{\leq\aleph_0}$ there is $I \in [\kappa]^{\aleph_1}$ such that

- $\mathrm{cf}(I) = \omega_1$;
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A Characterization of FRP

Theorem 6 (F., Sakai, Soukup and Usuba)

FRP $\Leftrightarrow$ *For any regular* $\mu > \aleph_1$, *there is no almost essentially disjoint ladder system* $g : E \to [\mu]^{\aleph_0}$ *for any stationary* $E \subseteq E^\mu_\omega$.

- $E^\mu_\omega = \{\alpha < \mu : \mathrm{cf}(\alpha) = \omega\}$.
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"$\Leftarrow$": Suppose that $\neg\mathrm{FRP}$ and let λ^* be the least regular cardinal s.t. $\neg\mathrm{FRP}(\lambda^*)$. Then there are a stationary $E \subseteq E^{\lambda^*}_{\omega}$ and a ladder system $g : E \to [\lambda^*]^{\aleph_0}$ s.t. , for any $I \in [\lambda^*]^{\aleph_1}$ with $\mathrm{cf}(I) = \omega_1$ and closed with respect to g, we have:

$$Z_I = \{x \in [I]^{\aleph_0} \ : \ \sup(x) \in E \cap I \text{ and } g(\sup(x)) \subseteq x\}$$

is non-stationary in $[I]^{\aleph_0}$.

Let $\sigma : \lambda^* \to {}^{\aleph_0 >}\lambda^*$ be a λ^* to 1 surjection and $C^* = \{\alpha < \lambda^* \ :$ for all $a \in {}^{\aleph_0 >}\alpha$, $\{\gamma < \alpha \ : \ \sigma(\gamma) = a\}$ is cofinal in $\alpha\}$. Then C^* is a club subset of λ^*. Thus $E^* = E \cap C^*$ is a stationary subset of λ^*.

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FRP $\Leftrightarrow$ *For any regular* $\mu > \aleph_1$*, there is no almost essentially disjoint ladder system* $g : E \rightarrow [\mu]^{\aleph_0}$ *for any stationary* $E \subseteq E^{\mu}_{\omega}$.

Sketch of the proof. "$\Rightarrow$": Easy.

"$\Leftarrow$": Suppose that $\neg$FRP and let λ^* be the least regular cardinal s.t. $\neg\mathrm{FRP}(\lambda^*)$. Then there are a stationary $E \subseteq E^{\lambda^*}_{\omega}$ and a ladder system $g : E \rightarrow [\lambda^*]^{\aleph_0}$ s.t. , for any $I \in [\lambda^*]^{\aleph_1}$ with $\mathrm{cf}(I) = \omega_1$ and closed with respect to g, we have:

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is non-stationary in $[I]^{\aleph_0}$.

Let $\sigma : \lambda^* \rightarrow {}^{\aleph_0>}\lambda^*$ be a λ^* to 1 surjection and $C^* = \{\alpha < \lambda^* :$ for all $a \in {}^{\aleph_0>}\alpha$, $\{\gamma < \alpha : \sigma(\gamma) = a\}$ is cofinal in $\alpha\}$.Then C^* is a club subset of λ^*. Thus $E^* = E \cap C^*$ is a stationary subset of λ^*.

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"Mathematical" characterizations of FRP

Using Theorem 6, we can show that most of the reflection theorems mentioned before are actually equivalent to FRP over ZF.

Corollary 7

FRP is equivalent to each of the following assertions over ZFC:

(A) *For every locally separable countably tight topological space X, if X is not meta-Lindelöf, then there is a subspace of X of cardinality $\leq \aleph_1$ which is not meta-Lindelöf.*

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Axiom R-like extension of Fodor-type reflection principle (FRP^R)

$\mathrm{FRP}^R(\kappa)$: For any ω_1-club $\mathcal{T} \subseteq [\kappa]^{\aleph_1}$, stationary $S \subseteq E^\kappa_\omega$ and mapping $g : S \to [\kappa]^{\leq\aleph_0}$ there is $I \in \mathcal{T}$ such that

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- The proof of $\mathrm{RP} \Rightarrow \mathrm{FRP}$ can be modified to prove $\mathrm{Axiom\ R} \Rightarrow \mathrm{FRP}^R$.

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Conjecture. FRP^R in the theorem above can be replaced by FRP.

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FRP and its “mathematical” characterizations (18/18)

Ich danke Ihnen für Ihre Aufmersamkeit.