Beautiful Theorems in Mathematics

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The beauty in mathematics ?

- Mathematics is divided into many areas (Algebra, Geometry, Analysis, etc.). Each of these "areas" is a world of its own. For mathematicians coming from different areas of mathematics it is often very difficult to understand what the others are doing in their research.

- The conceptions of the beauty of mathematics might be also quite diverse for mathematicians from different fields.

- Nevertheless all mathematicians are tightly connected to each other. This can be seen, for example, in terms of "collaboration distance":

AMS MathSciNet: Collaboration Distance

The beauty in mathematics !

- Many mathematicians consider the beauty as one of the most important factors in mathematics — you can find may books, articles in internet, YouTube clips etc. in which mathematicians, prominent or rather unknown, declare this.

For example:

Interviews at the School of Mathematics of University of Bristol, England

The beauty in mathematics ! (2/2)

- The following are three possible types of situations in which you may find the beauty in mathematics:

— We often detect the beauty in mathematics when:

▷ a seminal idea is formulated in a simple statement with a short elegant proof.

An example: Pythagorean Theorem

▷ a complex and exact theory is developed combining many ideas coming from different areas of mathematics to obtain a marvelous result.

Examples: ???

▷ a result is unexpected and surprising but you have to accept it as a true theorem because the proof is correct.

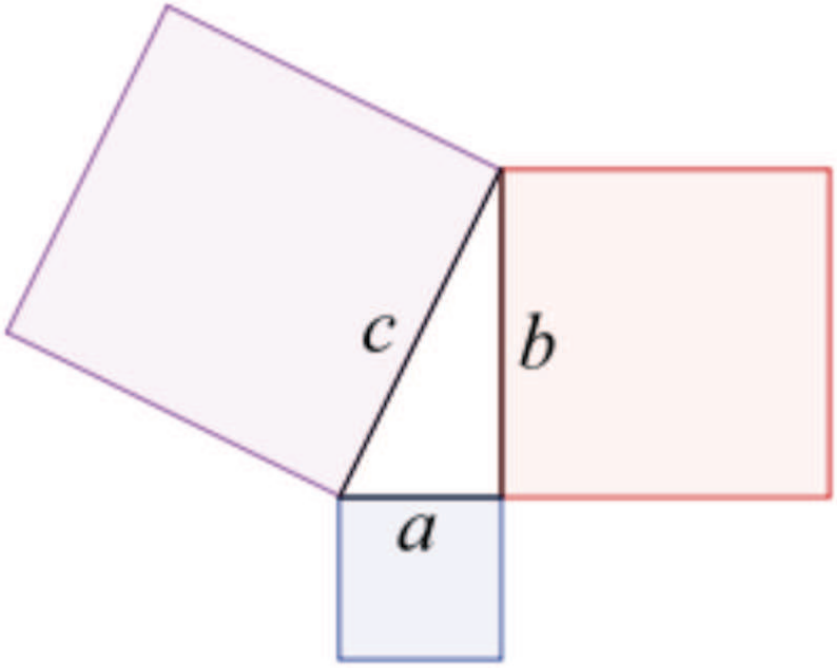
An example: 20° cannot be constructed!

References

- 渕野 昌，美は一本の毛で男をひっぱるだろう， 現代思想 2017 年 3 月臨時増刊号，Vol.45-5, 総特集＝知のトップランナー 50 人の美しいセオリー，102–108, (2017).
- Emil Artin, Galois Theory. Dover Publications (1998). ISBN 0-486-62342-4. (Reprinting of second revised edition of 1944, The University of Notre Dame Press).

Thank you for your attention.

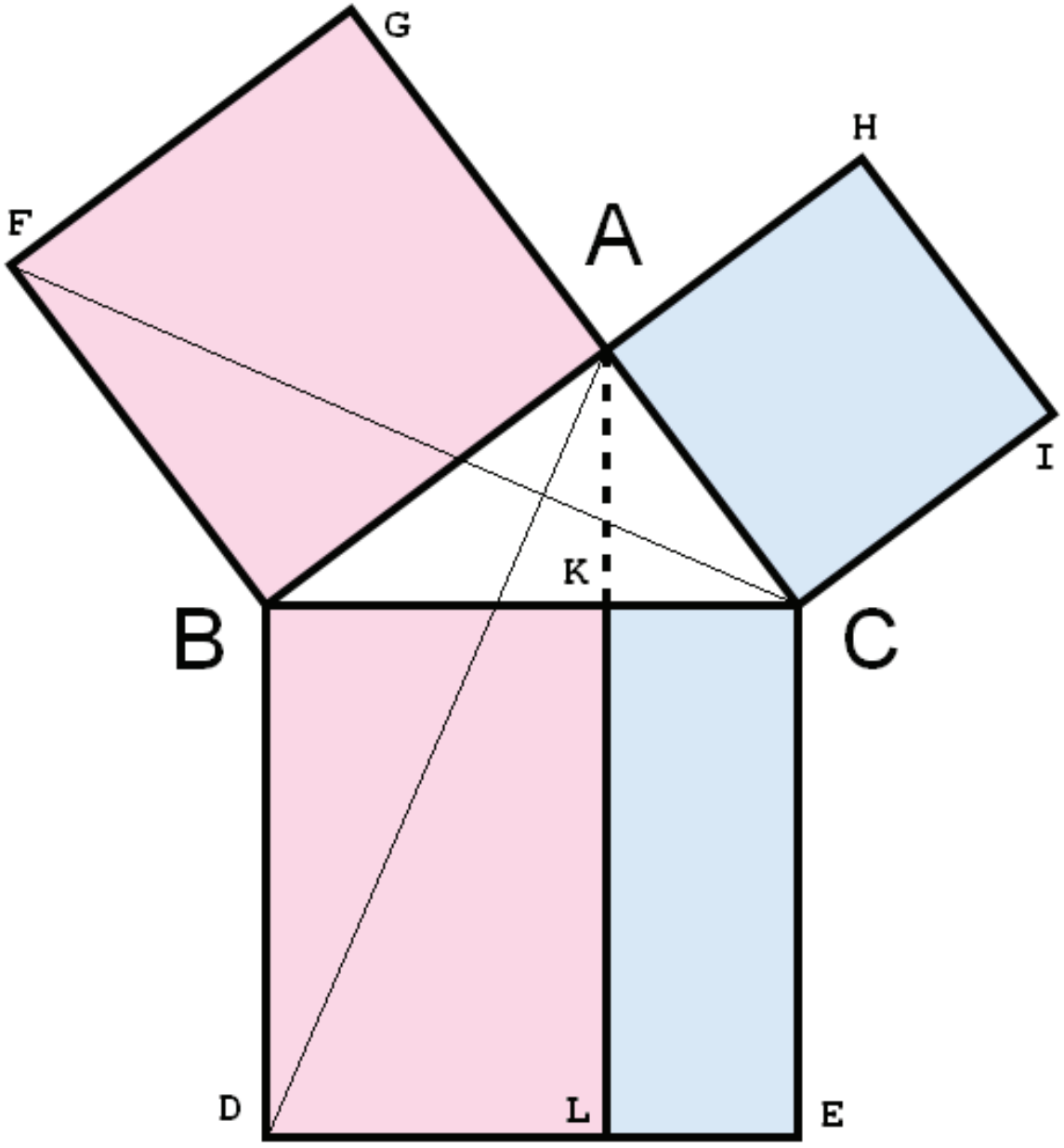
Pythagorean Theorem



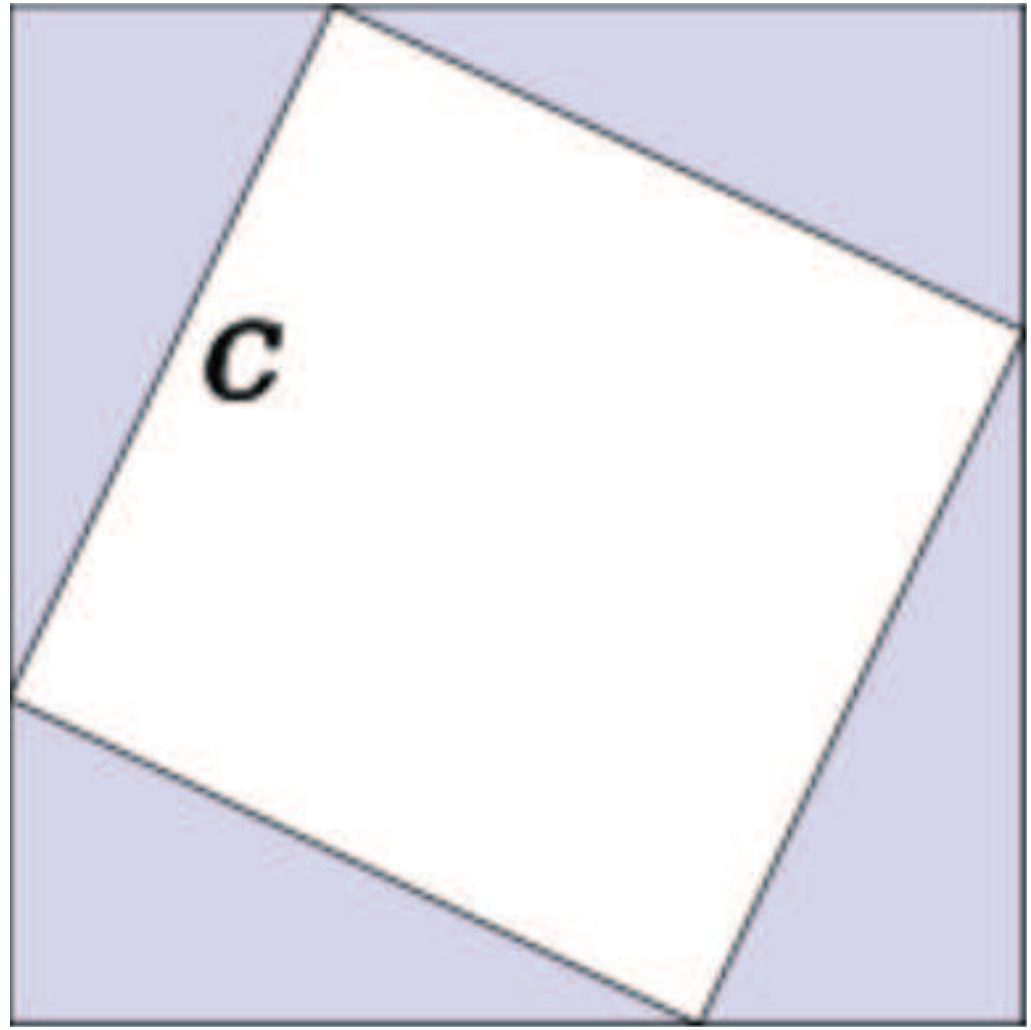
Theorem 1

Let T be an arbitrary right angled triangle with sides of length a, b, c where c is the length of the hypotenuse (斜辺 in Japanese). Then we have $a^2 + b^2 = c^2$.

Euclid's Proof



Another Proof



$$4 \times \frac{1}{2}ab + c^2 = (a+b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow \quad c^2 = a^2 + b^2$$

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Beauty in complex and exact theories

- It is difficult to give an example of such a theory: Either you already know very well the theory or I give a one-semester course on the theory and at the end of the semester you will begin to appreciate the beauty of the theory.

Beauty in complex and exact theories (2/2)

- Some of my favorite beautiful theories:

  - The proof of Fermat's Last Theorem
    (I know only a very rough idea of how it is done but I do not know the details of the proof)
  - the proof of Robertson-Seymour Theorem of graph theory
    (This is one of the most complicated proofs in the history of mathematics. It is one of my dreams to create a new proof of mine using methods from mathematical logic.)
  - PCF theory of Saharon Shelah
    (inaccessible for almost everybody except for some specialists)
  - Galois theory
    (anybody with university level math general education can learn it)

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Angle Trisection

- Using the Galois Theory we can prove that:

  The angle of 20° is not constructible (by using only straightedge and compass)

- In contrast to the non-constructibility above we *can* construct 60° (just draw an equilateral triangle!)

- Thus we obtain a negative answer to the long-standing open problem of ancient Greek mathematics:

  Is there a method (algorithm) only using straightedge and compass with which any given angle can be trisectioned.

- Note that there is a method to devide any angle into two equal angles.

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