

# Beautiful Theorems in Mathematics

Sakaé Fuchino (渚野 昌)

Graduate School of System Informatics  
Kobe University

(神戸大学大学院 システム情報学研究科)

<http://fuchino.ddo.jp/index-e.html>

(April 27, 2017 (15:59 JST) version)

**25 April 2017**

**13th Hyogo Science E-cafe  
NESCAFÉ Sannomiya**

This presentation is typeset by p<sup>A</sup>T<sub>E</sub>X with beamer class.

The printer version of these slides is going to be downloadable as

<http://fuchino.ddo.jp/slides/kobe-science-cafe-17-04-25-pf.pdf>

# The beauty in mathematics ?

- ▶ Mathematics is divided into many areas (Algebra, Geometry, Analysis, etc.). Each of these "areas" is a world of its own. For mathematicians coming from different areas of mathematics it is often very difficult to understand what the others are doing in their research.
- ▶ The conceptions of the beauty of mathematics might be also quite diverse for mathematicians from different fields.
- ▶ Nevertheless all mathematicians are tightly connected to each other. This can be seen, for example, in terms of "collaboration distance":

AMS MathSciNet: Collaboration Distance

# The beauty in mathematics !

- ▶ Many mathematicians consider the beauty as one of the most important factors in mathematics — you can find many books, articles in internet, YouTube clips etc. in which mathematicians, prominent or rather unknown, declare this.

For example:

Interviews at the School of Mathematics of University of Bristol, England



## The beauty in mathematics ! (2/2)

- ▶ The following are three possible types of situations in which you may find the beauty in mathematics:

— We often detect the beauty in mathematics when:

- ▷ a seminal idea is formulated in a simple statement with a short elegant proof.  
An example: Pythagorean Theorem
- ▷ a complex and exact theory is developed combining many ideas coming from different areas of mathematics to obtain a marvelous result.  
Examples: ???
- ▷ a result is unexpected and surprising but you have to accept it as a true theorem because the proof is correct.

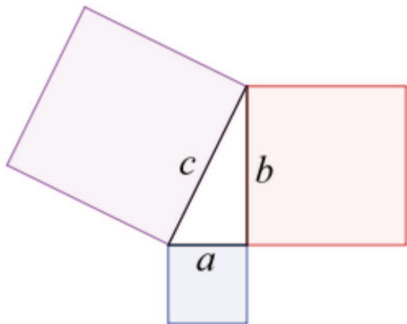
An example:  $20^\circ$  cannot be constructed!

-  渕野 昌, 美は一本の毛で男をひつぱるだろう, 現代思想 2017年3月臨時増刊号, Vol.45-5, 総特集=知のトップランナー 50人の美しいセオリー, 102-108, (2017).
-  Emil Artin, Galois Theory. Dover Publications (1998). ISBN 0-486-62342-4. (Reprinting of second revised edition of 1944, The University of Notre Dame Press).

Thank you for your attention.



# Pythagorean Theorem

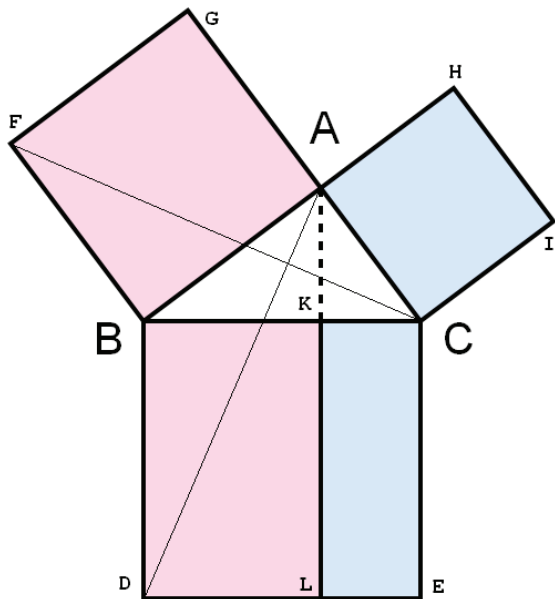


## Theorem 1

Let  $T$  be an arbitrary right angled triangle with sides of length  $a$ ,  $b$ ,  $c$  where  $c$  is the length of the hypotenuse (斜辺 in Japanese).

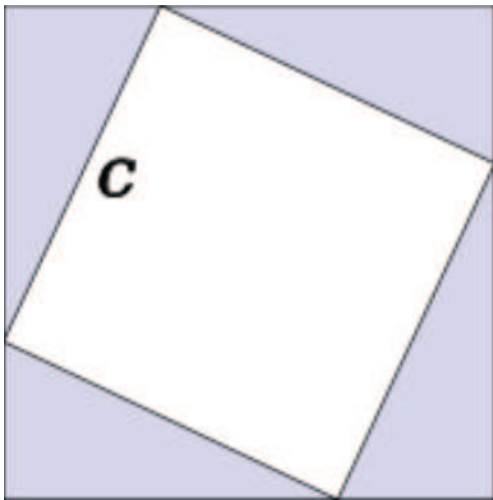
Then we have  $a^2 + b^2 = c^2$ .

## Euclid's Proof





## Another Proof



$$4 \times \frac{1}{2}ab + c^2 = (a + b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow c^2 = a^2 + b^2$$

もどる

## Beauty in complex and exact theories

- ▶ It is difficult to give an example of such a theory: Either you already know very well the theory or I give a one-semester course on the theory and at the end of the semester you will begin to appreciate the beauty of the theory.

## Beauty in complex and exact theories (2/2)

- ▶ Some of my favorite beautiful theories:
  - ▷ The proof of Fermat's Last Theorem  
(I know only a very rough idea of how it is done but I do not know the details of the proof)
  - ▷ the proof of Robertson-Seymour Theorem of graph theory  
(This is one of the most complicated proofs in the history of mathematics. It is one of my dreams to create a new proof of mine using methods from mathematical logic.)
  - ▷ PCF theory of Saharon Shelah  
(inaccessible for almost everybody except for some specialists)
  - ▷ Galois theory  
(anybody with university level math general education can learn it)

## Angle Trisection

- ▶ Using the **Galois Theory** we can prove that:

The angle of  $20^\circ$  is not constructible (by using only straightedge and compass)

- ▷ In contrast to the non-constructibility above we *can* construct  $60^\circ$  (just draw an equilateral triangle!)

- ▶ Thus we obtain a negative answer to the long-standing open problem of ancient Greek mathematics:

Is there a method (algorithm) only using straightedge and compass with which any given angle can be trisected.

- ▷ Note that there is a method to divide any angle into to equal angles.

もどる