Thm 11.3 (A variant of Dow's Metrization Thm) by Z. Balogh

If X is ~~regular~~ countably compact (Hausdorff) and if all $Y \in [X]^{<\aleph_2}$ have point countable open base then X is metrizable.

Cor 11.4 (Dow's Metrization Thm)

If X is ~~regular and~~ countably compact Hausdorff and if all subspaces $Y \in [X]^{<\aleph_2}$ are metrizable, then X is metrizable.

Sakaé Fuchino,
Fodor-type Reflection Principle and
Balogh's reflection theorems

http://fuchino.ddo.jp/papers/balogh-x.pdf

の §2 を参照.

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Uruba, 2010

(*) For all top space X if all $Y \in [X]^{<\aleph_2}$ are metrizable then X is metrizable. (false)

(**) For all first countable X if all $Y \in [X]^{<\aleph_2}$ are metrizable then X is metrizable.

We know $V=L$ negates (**)

Consistency of (**) is open (Hamburger's Problem)

(***) For all locally countably compact X if all $Y \in [X]^{<\aleph_2}$ are metrizable then X is metrizable.

We know $V=L$ still negates (***)

If ZFC + $\exists$ strongly compact cardinal is consistent then so is ZFC + (***).

(*) is false:

Let κ be any cardinal with $\underbrace{cf(\kappa)}_{\delta} > \omega_1$

Let $\langle \alpha_\xi \mid \xi < \delta \rangle$ be a strictly increasing sequence of ordinals below κ s.t. $\lim_{\xi < \delta} \alpha_\xi = \kappa$

$X = \kappa + 1 = \kappa \cup \{\kappa\}$

with the topology generated by

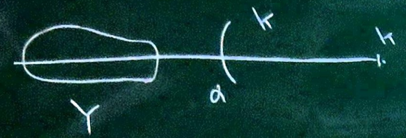
$\{\{\alpha\} \mid \alpha < \kappa\} \cup \{O_\alpha \mid \alpha < \kappa\}$ where

$O_\alpha = \{\alpha_\xi \mid \xi < \delta\} \cup \{\kappa\} \setminus \alpha$

X is not metrizable since $\chi(\kappa, X) = \delta > \omega$

But all $Y \in [X]^{<\delta}$ are metrizable

since they are discrete



$O_\alpha \cap Y = \{\kappa\}$

- Discrete spaces are metrizable

[Define $d(x,y) = 1$ for all $x, y \in X$, $x \neq y$]