Linear Algebra 2 Expected/possible questions in the final exam

担当 (Lecturer): 渕野 昌 (Sakaé Fuchino)

2nd quarter 2019 (2019/07/24 23:38version)

This list might be further edited/extended until Aug. 1. In particular, some hints can be added at some monent (this applies also to the questions in the exercise of July 10).

In the final exam some of the questions similar/closely connected to the questions below as well as questions in the exercise of July 10 (`http://fuchino.ddo.jp/kobe/lin-alg-2-e-2019-2q-uebung.pdf`) will be asked. There might be also one or tow additional questions in the final exam, which might be slightly more challenging.

The present page is downloadable as `http://fuchino.ddo.jp/kobe/lin-alg-1-2-e-2q-pre-final.tex`

I. Calculate the following:

$$\begin{vmatrix} 3 & 1 & 0 & 0 & 0 \\ 6 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 2 & 3 \end{vmatrix} \qquad \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{vmatrix} \qquad \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 3 & 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 4 & 0 \\ 5 & 0 & 0 & 0 & 5 \end{vmatrix}$$

II. Prove the following claims:

(a) For an $n \times n$ matrix A, if A is invertible then $\det(A) \neq 0$ and $\det(A^{-1}) = (\det(A))^{-1}$.

(b) For any $n \times n$ matrices A and P, if P is invertible then $\det(A) = \det(P^{-1}AP)$.

(c) If an $n \times n$ matrix $A = [a_{ij}]$ is an upper trianglar matrix (that is such a matrix that $a_{i,j} = 0$ for all $1 \leq j < i$), then $\det(A) = \prod_{i=1}^{n} a_{i,i}$.

III. Calculate the determinant of matrices of basic transformations. Use this to show that an $n \times n$ matrix A has non-zero determinant if and only if A is invertible.