# Linear Algebra I  Exercise on June 21

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This sheet of exercises (and its possible update) is downloadable as

`http://kurt.scitec.kobe-u.ac.jp/~fuchino/kobe/linalg-I-ss12-exercise.pdf`

**1.** Determine if the vectors in the following (a) — (d) are linearly independent. Justify each answer.

(a) $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}$  (b) $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -8 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix}$  (d) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -9 \end{bmatrix}$

**2.** Determine if the linear transformations with the standard matrix $A$ in the following (a) and (b) is one to one.

(a) $A = \begin{bmatrix} 0 & -3 & 9 \\ 2 & 1 & -7 \\ -1 & 4 & -5 \\ 1 & -4 & 2 \end{bmatrix}$  (b) $A = \begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$

**3.** Mark each of the following statements True or False. Justify each answer.

(a) The columns of a matrix $A$ are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution.

(b) If $S$ is a linearly dependent set, then each vector is linear combination of the other vectors in $S$.

(c) The columns of any $4 \times 5$ matrix are linearly dependent.

(d) If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent then $\mathbf{z}$ is in $\mathrm{Span}\{\mathbf{x}, \mathbf{y}\}$.

(e) If $\mathbf{u}$ and $\mathbf{v}$ are linearly independent, and if $\mathbf{w}$ is in $\mathrm{Span}\{\mathbf{u}, \mathbf{v}\}$ then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent.

**3.** In the following (a) and (b) find a vector $\mathbf{x}$ which satisfies $\varphi_A(\mathbf{x}) = \mathbf{b}$

(a) $A = \begin{bmatrix} 1 & 0 & -3 \\ -3 & 1 & 6 \\ 2 & -2 & -1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -3 \\ 2 & -5 & 6 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -6 \\ -4 \\ -5 \end{bmatrix}$

**4.** Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = mx + b$.

(a) Show that $f$ is a linear transformation when $b = 0$.

(b) Find a property of a linear transformation that is violated when $b \neq 0$.

**5.** Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly dependent set in $\mathbb{R}^n$. Explain why the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly dependent.

**6.** Find the standard matrices of the following linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$:

(a) Reflection through the $x_2$-axis.

(b) Reflection through the origin.

(c) Horizontal expansion by factor $k$ (i.e. $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} kx \\ y \end{bmatrix}$)

$\boxed{\textbf{7.}}$ Find the standard matrix of the linear transformation

$$\varphi : \mathbb{R}^2 \to \mathbb{R}^3; \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} 3x_1 + x_2 \\ 5x_1 + 7x_2 \\ x_1 + 3x_2 \end{bmatrix}.$$

$\boxed{\textbf{8.}}$ Show that, for a linear transformation $\varphi$, its standard matrix is determined uniquely, that is, if $\varphi : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and $A$ and $B$ are matrices such that $A\mathbf{x} = \varphi(\mathbf{x}) = B\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^n$ then we have $A = B$.