Some more remarks on Riis' Axiom

a joint work with Diego Alejandro Mejía (メヒア ディエゴ)

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Recent Developments in Set Theory of the Reals

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Sakaé Fuchino **(渕野 昌)**

- The following is a report about (my) progress in an ongoing joint research with Diego Mejia.
- The talk is a continuation of one of the topics in a talk at the set-theory (virtual) workshop in Kobe on March 9, 2021.

The research is supported by
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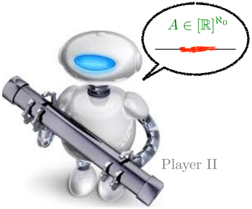
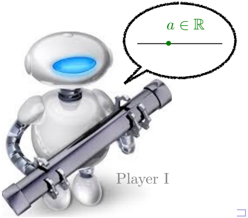
Riis' Axiom — The guessing game

- $\mathcal{N}$:= the ideal of null sets $\subseteq \mathbb{R}$.
- We consider the following guessing game between Player I and Player II: Player I guesses a real $a \in \mathbb{R}$; simultaneously, Player II guesses a countable set $A \in [\mathbb{R}]^{\aleph_0}$.
- Player II wins, if $a \in A$.
- A sequence $\langle A_r : r \in \mathbb{R} \rangle$ of countable sets is called a Monte Carlo strategy of Player II if, for any $a \in \mathbb{R}$,

$$\{r \in \mathbb{R} : a \notin A_r\} \in \mathcal{N}.$$



- Player II wins the game as above with the "probability 1", if it chooses a real $r \in \mathbb{R}$ randomly and take A_r as its move.



Riis' Axiom

- Søren Riis thought that, since countable sets should be negligible compared to the continuum, it should be impossible that Player II has such a strategy in the game and formulated:

> **(Riis' Axiom [Riis])** There is no Monte Carlo st. for Player II in the game as on the previous slide.

- Riis' Axiom has several interesting consequences, for example:

Theorem 1. (Riis' Axiom) CH does not hold.

Proof. Suppose CH holds. Let $\langle I_\alpha : \alpha \in \omega_1 \rangle$ be a filtration of $\mathbb{R}$. Let $\iota : \mathbb{R} \to \omega_1$ a bijection.

▷ For $r \in \mathbb{R}$, let $A_r := I_{\iota(r)}$. Then $\langle A_r : r \in \mathbb{R} \rangle$ is a Monte Carlo st. for Player II in the game. □

[Riis] Søren Riis, FOM: A proof of not-CH, Sun Sep 13 12:24:49 EDT (1998).

Riis' Axiom — A more general setting

- For ideals $I, J \subseteq \mathcal{P}(\mathbb{R})$,

(R_I^J): There is a sequence $\langle A_r : r \in \mathbb{R}\rangle$ of elements of J s.t., for any $a \in \mathbb{R}$, we have $\{r \in \mathbb{R} : a \notin A_r\} \in I$.

- We write "$<\kappa$" to denote the ideal $[\mathbb{R}]^{<\kappa}$; $\mathcal{N} :=$ the ideal of null sets $\subseteq \mathbb{R}$. With this notation

Riis' Axiom $\Leftrightarrow$ $\neg\, \mathrm{R}_{\mathcal{N}}^{<\aleph_1}$.

- ▷ We shall call $\langle A_r : r \in \mathbb{R}\rangle$ in the statement of R_I^J a Monte Carlo st. for (I, J), and $\{r \in \mathbb{R} : a \notin A_r\}$ the set of exceptions.

- R_I^J can be also defined similarly for ideals I, J over an arbitrary infinite set X (instead of over $\mathbb{R}$).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Tukey connections on triples (relational systems)

The following framework which also appeared in the tutorial lectures and some other talks as well was introduced by Peter Vojtáš at beginning of 1990's and further developed by people including Andreas Blass and Tomek Bartoszyński.

- Let

  $\mathcal{T} := \{\mathbb{A} : \mathbb{A} = \langle A_-, A_+, A\rangle,\ A_-, A_+ \text{ non empty sets},\ A \subseteq A_- \times A_+\}.$

- ▷ For $\mathbb{A}$, $\mathbb{B} \in \mathcal{T}$ with $\mathbb{A} = \langle A_-, A_+, A\rangle$ and $\mathbb{B} = \langle B_-, B_+, B\rangle$,

  $\mathbb{A} \leq_{\mathcal{T}} \mathbb{B}$:⇔ there is $\varphi = \langle \varphi_-, \varphi_+\rangle$ s.t. $\varphi_- : A_- \to B_-$, $\varphi_+ : B_+ \to A_+$, for any $a \in A_-$, $b \in B_+$, $\big(\varphi_-(a)\ B\ b \ \Rightarrow\ a\ A\ \varphi_+(b)\big)$.

$$\begin{tikzcd}
A_- \arrow[rrr, bend left, "\varphi_-"] & A_+ & & B_- & B_+ \arrow[lll, bend left, "\varphi_+"]
\end{tikzcd}$$

$$a\ A\ \varphi_+(b) \qquad \Leftarrow \qquad \varphi_-(a)\ B\ b$$

- If $\mathbb{A} \leq_{\mathcal{T}} \mathbb{B}$, and $\varphi = \langle \varphi_-, \varphi_+\rangle$ is as in the definition of $\leq_{\mathcal{T}}$, we shall write $\mathbb{A} \leq^{\varphi}_{\mathcal{T}} \mathbb{B}$ or $\mathbb{A} \leq^{\varphi_-,\varphi_+}_{\mathcal{T}} \mathbb{B}$.

Tukey connections on triples 2/5

- We can also consider Tukey relation $\leq_{\mathcal{T}}$ restricted to some appropriate subclass $\mathcal{T}'$ of $\mathcal{T}$.

- If we define $\mathcal{T}' := \{\langle A_-, A_+, A\rangle \in \mathcal{T} \,:\, A_- = A_+\}$ and identify structures $\langle X, R\rangle$ with binary relation R with the triple $\langle X, X, R\rangle$, the class structure $(\mathcal{T}', \leq_{\mathcal{T}})$ corresponds to the classical Galois-Tukey connection of binary relations.

Tukey connections on triples 3/5

Lemma 1. $\le_{\mathcal{T}}$ is transitive.

Proof. If $\mathbb{A} \le_{\mathcal{T}}^{\varphi_-,\varphi_+} \mathbb{B}$ and $\mathbb{B} \le_{\mathcal{T}}^{\psi_-,\psi_+} \mathbb{C}$ then $\mathbb{A} \le_{\mathcal{T}}^{\psi_-\circ\varphi_-,\varphi_+\circ\psi_+} \mathbb{C}$. □

- For $\mathbb{A} \in \mathcal{T}$ with $\mathbb{A} = \langle A_-, A_+, A\rangle$, let $\mathbb{A}^\perp := \langle A_+, A_-, A^\perp\rangle$ where $A^\perp := \{\langle y, x\rangle \in A_+ \times A_- \ : \ x \not{A}\, y\}$.

Lemma 2. (1) $(\mathbb{A}^\perp)^\perp = \mathbb{A}$.

(2) $\mathbb{A} \le_{\mathcal{T}} \mathbb{B} \ \Leftrightarrow\ \mathbb{B}^\perp \le_{\mathcal{T}} \mathbb{A}^\perp$.

Proof. (1) is clear by the definition of “$(\cdot)^\perp$”.

(2):
$$\begin{aligned}\mathbb{A} \le_{\mathcal{T}}^{\varphi_-,\varphi_+} \mathbb{B} \quad &\Leftrightarrow \quad \forall a \in A_- \forall b \in B_+ \Big(\varphi_-(a)\ B\ b \ \Rightarrow\ a\ A\ \varphi_+(b)\Big)\\ &\Leftrightarrow \quad \forall b \in B_+ \forall a \in A_- \Big(a \not{A}\ \varphi_+(b) \ \Rightarrow\ \varphi_-(a) \not{B}\ b\Big)\\ &\Leftrightarrow \quad \mathbb{B}^\perp \le_{\mathcal{T}}^{\varphi_+,\varphi_-} \mathbb{A}^\perp\end{aligned}$$
□

back to the proof of Lemma 4.

back to the proof of Corollary 7.

Tukey connections on triples 4/5

- Suppose $\mathbb{A} \in \mathcal{T}$ with $\mathbb{A} = \langle A_-, A_+, A \rangle$, let

  $\mathfrak{d}(\mathbb{A}) := \{D \in \mathcal{P}(A_+) \,:\, \forall a \in A_- \exists d \in D \ (a \ A \ d)\}$,

  $\mathfrak{b}(\mathbb{A}) := \{B \in \mathcal{P}(A_-) \,:\, \forall d \in A_+ \exists b \in B \ (b \not{A} \ d)\}$

  and

  $\mathfrak{d}_{\mathbb{A}} := \min\{|\,D\,| \,:\, D \in \mathfrak{d}(\mathbb{A})\}$,

  $\mathfrak{b}_{\mathbb{A}} := \min\{|\,B\,| \,:\, B \in \mathfrak{b}(\mathbb{A})\}$

Lemma 3. $\mathfrak{d}_{\mathbb{A}} = \mathfrak{b}_{\mathbb{A}^\perp}$ and $\mathfrak{b}_{\mathbb{A}} = \mathfrak{d}_{\mathbb{A}^\perp}$

Proof.

$$\begin{aligned} D \in \mathfrak{d}(\mathbb{A}) \ &\Leftrightarrow \ \forall a \in A_- \exists d \in D \ (a \ A \ d) \\ &\Leftrightarrow \ \forall a \in A_- \exists d \in D \ (d \ \not{A}^\perp \ a) \\ &\Leftrightarrow \ D \in \mathfrak{b}(\mathbb{A}^\perp). \end{aligned}$$

Thus $\mathfrak{d}(\mathbb{A}) = \mathfrak{b}(\mathbb{A}^\perp)$ and $\mathfrak{d}_{\mathbb{A}} = \mathfrak{b}_{\mathbb{A}^\perp}$.

The second equation is proved similarly by showing $\mathfrak{b}(\mathbb{A}) = \mathfrak{d}(\mathbb{A}^\perp)$. ⧉

back to the proof of Lemma 4.

Tukey connections on triples 5/5

Lemma 4. $\mathbb{A} \leq_{\mathcal{T}} \mathbb{B} \;\Rightarrow\; \mathfrak{d}_{\mathbb{A}} \leq \mathfrak{d}_{\mathbb{B}}$ and $\mathfrak{b}_{\mathbb{A}} \geq \mathfrak{b}_{\mathbb{B}}$.

Proof. Assume $\mathbb{A} \leq_{\mathcal{T}}^{\varphi_-,\varphi_+} \mathbb{B}$ holds. We show:

Claim. $D \in \mathfrak{d}(\mathbb{B}) \;\Rightarrow\; \varphi_+{}''D \in \mathfrak{d}(\mathbb{A})$.

$\vdash$ Suppose $D \in \mathfrak{d}(\mathbb{B})$. Then, for any $a \in A_-$, there is $d \in D$ with $\varphi_-(a)\ B\ d$. It follows, by the definition of $\leq_{\mathcal{T}}$, that $a\ A\ \underbrace{\varphi_+(d)}_{\in \varphi_+{}''D}$. This shows that $\varphi_+{}''D \in \mathfrak{d}(\mathbb{A})$. $\dashv$

Let $D \in \mathfrak{d}(\mathbb{B})$ be s.t. $|\,D\,| = \mathfrak{d}_{\mathbb{B}}$. Then, we have,

$$\mathfrak{d}_{\mathbb{A}} \underbrace{\leq}_{\text{by Claim above.}} |\,\varphi_+{}''D\,| \leq |\,D\,| = \mathfrak{d}_{\mathbb{B}}.$$

For the second inequation, since we have $\mathbb{B}^{\perp} \leq_{\mathcal{T}} \mathbb{A}^{\perp}$ by Lemma 2 . It follows that $\mathfrak{d}_{\mathbb{B}^{\perp}} \leq \mathfrak{d}_{\mathbb{A}^{\perp}}$ by above.

Thus $\mathfrak{b}_{\mathbb{B}} \leq \mathfrak{b}_{\mathbb{A}}$ by Lemma 3 .

back to the prf. of Cor. 7

Cardinal invariants of ideals

- Let X be an infinite set and I an ideal over X.
- ▷ the usual cardinal invariants of the ideal I can be represented as dominating and bounding numbers of triples:

Lemma 5. (1) $\mathrm{cof}(I) = \mathfrak{d}_{\langle I, I, \subseteq \rangle}$,

(2) $\mathrm{add}(I) = \mathfrak{b}_{\langle I, I, \subseteq \rangle} = \mathfrak{d}_{\langle I, I, \subseteq \rangle^\perp} = \mathfrak{d}_{\langle I, I, \not\supseteq \rangle}$,

(3) $\mathrm{cov}(I) = \mathfrak{d}_{\langle X, I, \in \rangle}$

(4) $\mathrm{non}(I) = \mathfrak{b}_{\langle X, I, \in \rangle} = \mathfrak{d}_{\langle X, I, \in \rangle^\perp} = \mathfrak{d}_{\langle I, X, \not\ni \rangle}$.

Proof. (3): $\mathfrak{d}(\langle X, I, \in \rangle) = \{D \subseteq I \,:\, \forall x \in X \; \exists s \in I \; (x \in s)\}$
$= \{D \subseteq I \,:\, \bigcup D = X)\}$. Thus $\mathfrak{d}_{\langle X, I, \in \rangle} = \mathrm{cov}(I)$.

(4): $\mathfrak{b}(\langle X, I, \in \rangle) = \{S \subseteq X \,:\, \forall s \in I \; \exists x \in S \; (x \notin s)\}$
$= \{S \subseteq X \,:\, S \notin I)\}$. Thus $\mathfrak{b}_{\langle X, I, \in \rangle} = \mathrm{non}(I)$.

$\mathfrak{b}_{\langle X, I, \in \rangle} = \mathfrak{d}_{\langle X, I, \in \rangle^\perp}$ follows from Lemma 3.

$\mathfrak{d}_{\langle X, I, \in \rangle^\perp} = \mathfrak{d}_{\langle I, X, \not\ni \rangle}$ from the definition of $\mathbb{A}^\perp$.

back to the prf. of Cor. 7

(1) and (2) are proved similarly. □

Riis' principles in terms of Tukey connection

Proposition 6. For ideals I, J over an infinite set set X, we have
$\mathrm{R}_I^J \;\Leftrightarrow\; \langle X, I, \in\rangle \leq_{\mathcal{T}} \langle X, J, \in\rangle^{\perp}$ $(= \langle J, X, \not\ni\rangle)$.

Proof. $\Rightarrow$: Suppose R_I^J holds. This means that there is a sequence $\langle E_x : x \in X\rangle$ in J s.t., for any $y \in X$, $\{x \in X : E_x \not\ni y\} \in I$.

▷ Let $\varphi_- : X \to J$; $x \mapsto E_x$ and
$\varphi_+ : X \to I$; $y \mapsto D_y := \{x \in X : E_x \not\ni y\}$.

▷ For x, $y \in X$, $\varphi_-(x) \not\ni y \;\;\Leftrightarrow\;\; E_x \not\ni y \;\;\underset{\Leftrightarrow}{\Rightarrow}\;\; x \in D_y \;\;\Leftrightarrow\;\; x \in \varphi_+(y)$.

This shows that $\langle X, I, \in\rangle \leq_{\mathcal{T}}^{\varphi_-,\varphi_+} \langle X, J, \in\rangle^{\perp}$.

$\Leftarrow$: Suppose $\langle X, I, \in\rangle \leq_{\mathcal{T}}^{\varphi_-,\varphi_+} \langle X, J, \in\rangle^{\perp}$. Then, for $x, y \in X$, we have $\varphi_-(x) \not\ni y \Rightarrow x \in \varphi_+(y)$.

▷ Let $E_x := \varphi_-(x)$ for all $x \in X$. Then, for any $y \in X$,

$$\begin{aligned}\{x \in X : E_x \not\ni y\} &= \{x \in X : \varphi_-(x) \not\ni y\}\\ &\subseteq \{x \in X : x \in \varphi_+(y)\} = \varphi_+(y) \in I.\end{aligned}$$

Thus, $\langle E_x : x \in X\rangle$ witnesses R_I^J. □

Riis' principles in terms of Tukey connection (2/4)

Corollary 7. (1) For any ideals I, J over an infinite set X we have, $\mathrm{R}^J_I \Leftrightarrow \mathrm{R}^I_J$. (2) R^J_I implies $\mathrm{cov}(I) \leq \mathrm{non}(J)$ and $\mathrm{cov}(J) \leq \mathrm{non}(I)$.

Proof. $\mathrm{R}^J_I \underbrace{\Leftrightarrow}_{\text{by Proposition 6}} \langle X, I, \in\rangle \leq_{\mathcal{T}} \langle X, J, \in\rangle^{\perp} \underbrace{\Leftrightarrow}_{\text{by Lemma 2}} \langle X, J, \in\rangle \leq_{\mathcal{T}} \langle X, I, \in\rangle^{\perp}$

(1): $\underbrace{\Leftrightarrow}_{\text{by Proposition 6}} R^I_J.$

$\Downarrow$ Lemma 4

$$\mathrm{cov}(J) = \mathfrak{d}_{\langle X,J,\in\rangle} \leq \mathfrak{d}_{\langle X,I,\in\rangle^{\perp}} \underbrace{=}_{\text{by by Lemma 5}} \mathrm{non}(I)$$

$\Downarrow$ Lemma 4

(2):

$$\mathrm{cov}(I) \underbrace{=}_{\text{by Lemma 5}} \mathfrak{d}_{\langle X,I,\in\rangle} \leq \mathfrak{d}_{\langle X,J,\in\rangle^{\perp}} \underbrace{=}_{\text{by Lemma 5}} \mathrm{non}(J)$$

□

Riis' principles in terms of Tukey connection (3/4)

- Let $\mathcal{N}$:= null ideal over $\mathbb{R}$, $\mathcal{M}$:= meager ideal over $\mathbb{R}$.

Lemma 8. (Rothberger) $\mathrm{R}^{\mathcal{N}}_{\mathcal{M}}$ holds. Hence $\mathrm{R}^{\mathcal{M}}_{\mathcal{N}}$ also holds.

Proof. By Proposition 6 (and Corollary 7), it is enough to show $\langle \mathbb{R}, \mathcal{N}, \in \rangle \leq_{\mathcal{T}} \langle \mathcal{M}, \mathbb{R}, \not\ni \rangle$.

- Let $\mathbb{R} = \underbrace{N}_{\text{null set}} \dot\cup \overbrace{M}^{\text{meager set}}$. Let

$$\begin{tikzcd}
\mathbb{R} \arrow[rr, bend left, "\varphi_- : \quad a \mapsto a + M"] & \mathcal{N} & \mathcal{M} & \mathbb{R} \arrow[ll, bend left, "\varphi_+ : \quad b - N \leftarrow\!\shortmid b"]
\end{tikzcd}$$

- For a, $b \in \mathbb{R}$, $a + M \not\ni b \;\Leftrightarrow\; a + N \ni b \;\Rightarrow\; a \in b - N$.
This shows that $\langle \mathbb{R}, \mathcal{N}, \in \rangle \leq_{\mathcal{T}}^{\varphi_-, \varphi_+} \langle \mathcal{M}, \mathbb{R}, \not\ni \rangle$.

back to the prf. of Thm. 12

Riis' principles in terms of Tukey connection (4/4)

Remark 9. If $\mathrm{non}(I) = \mathrm{non}(J) = |X|$, then we have R_I^J.

Proof. Let $|X| = \mu$ and $X = \{x_\xi : \xi < \mu\}$. For ea ch $\xi < \mu$, let $E_\xi := \{x_\eta : \eta < \xi\}$. Then $\langle E_\xi : \xi < \mu\rangle$ reindexed by elements of X is a Monte-Carlo st. □

Corollary 10. Suppose that I is a (proper) ideal over X, $\kappa \leq |X|$ and $\mathrm{R}_I^{<\kappa}$ holds. Then $\mathrm{cov}(I) \leq \kappa$ and $|X| = non(I)$

Proof. By Corollary 7, $\mathrm{cov}([X]^{<\kappa}) = |X|$ and $\mathrm{non}([X]^{<\kappa}) = \kappa$. □

Independence of $R_{\mathcal{N}}^{<\aleph_1}$ over ZFC + ¬CH

Theorem 11. MA + ¬CH implies Riis' Axiom ($\neg R_{\mathcal{N}}^{<\aleph_1}$).

Proof. MA implies $\mathrm{cov}(\mathcal{N}) = 2^{\aleph_0}$. By Corollary 10, it follows that $\neg R_{\mathcal{N}}^{<\aleph_1}$. ☐

A set $L \subseteq \mathbb{R}$ is called a Luzin set, if for any uncountable $X \subseteq L$ is non-meager.

Theorem 12. (Yoshinobu, Recław-Zakrzewski) If there is a Luzin set of cardinality $2^{\aleph_0}$, then $R_{\mathcal{N}}^{<\aleph_1}$ holds.

Remark 13. If M is generic extension of ground model obtained more than continuum many (appropriate number of) Cohen reals, then, in M, the set of Cohen reals is a Lusin set of size $2^{\aleph_0}$.

Corollary 14. $R_{\mathcal{N}}^{<\aleph_1}$ is independent over ZFC + ¬CH.

Independence of $R_{\mathcal{N}}^{<\aleph_1}$ over ZFC + ¬CH (2/2)

Theorem 12. (Yoshinobu, Recław-Zakrzewski) If there is a Luzin set of cardinality $2^{\aleph_0}$, then $\mathrm{R}_{\mathcal{N}}^{<\aleph_1}$ holds.

- The proof is similar to the proof of Lemma 8 .

Proof. Let $L \subseteq \mathbb{R}$ be a Luzin set with $|L| = 2^{\aleph_0}$, and let $\mathbb{R} = \underbrace{N}_{\text{null set}} \dot{\cup} \overbrace{M}^{\text{meager set}}$.

- Let $L_N := L \cap N$ and $L_M = L \cap M$.
  Note that L_M is at most countable.
- Note: $\langle [L]^{<\aleph_1}, L, \not\ni \rangle \cong \langle [\mathbb{R}]^{<\aleph_1}, \mathbb{R}, \not\ni \rangle$. $\varphi_- : \quad a \mapsto a + L_M$
  We show $\langle \mathbb{R}, \mathcal{N}, \in \rangle \leq_{\mathcal{T}} \langle [L]^{<\aleph_1}, L, \not\ni \rangle$.
  - Let

$$\begin{tikzcd} \mathbb{R} \arrow[rr, bend left] & \mathcal{N} & \left[L\right]^{<\aleph_1} & L \arrow[ll, bend left] \end{tikzcd}$$

$\varphi_+ : \quad b - L_N \leftarrow\!\shortmid b$

- For a, $b \in \mathbb{R}$, $a + L_M \not\ni b \;\Leftrightarrow\; a + L_N \ni b \;\Rightarrow\; a \in b - L_N$.
  This shows that $\langle \mathbb{R}, \mathcal{N}, \in \rangle \leq_{\mathcal{T}}^{\varphi_-, \varphi_+} \langle [L]^{<\aleph_1}, L, \not\ni \rangle$.

Some work in progress

Lemma 8. (Rothberger) $\mathsf{R}_{\mathcal{M}}^{\mathcal{N}}$ holds. Hence $\mathsf{R}_{\mathcal{N}}^{\mathcal{M}}$ also holds.

Proposition A. $\mathsf{R}_{\mathcal{SN}}^{\mathcal{SM}}$ is independent.

Theorem 13. (Rothberger 1938) If there are Luzin set and Sierpinski set and at least one of then is of size $2^{\aleph_0}$ then CH holds.

Proposition A.(?) $\mathsf{R}_{\mathcal{M}}^{<\aleph_1} + \mathsf{R}_{\mathcal{N}}^{<\aleph_1} + \neg\mathsf{CH}$ is consistent.

Gracias por su atención.

ご清聴ありがとうございました.

Thank you for your attention!

Irek Recław† S.F. Peter Vojtáš

around 1992, in Wilmersdorf, Berlin

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