Abstract

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It is shown in [1] that the strong Löwenheim-Skolem theorems for stationary logics SDLS($\mathcal{L}^{\aleph_0}_{stat}$,$<\aleph_2$), SDLS$^-$($\mathcal{L}^{\aleph_0}_{stat}$,$<2^{\aleph_0}$), and SDLS$^{int}_+$($\mathcal{L}^{PKL}_{stat}$,$<2^{\aleph_0}$) decide the size of the continuum as $\aleph_1$, $\aleph_2$ and very large, respectively.

The Game Reflection Principle of Bernhard König (with the reflection point $<\aleph_2$) [2] implies SDLS($\mathcal{L}^{\aleph_0}_{stat}$,$<\aleph_2$) as well as Rado's Conjecture. König [2] proved that the Game Reflection Principle with the reflection point $<\aleph_2$ is equivalent to the statement that ω_2 is generically supercompact for σ-closed posets.

This suggests that we may regard (the existence of) generic large cardinals as ultimate reflection principles and ask if there are some natural notions of generic large cardinals which imply SDLS$^-$($\mathcal{L}^{\aleph_0}_{stat}$,$<2^{\aleph_0}$), and SDLS$^{int}_+$($\mathcal{L}^{PKL}_{stat}$,$<2^{\aleph_0}$), respectively.

The notion of Laver generically supercompact cardinals and its variations are introduced against this backdrop. In particular, The existence of a Laver generically supercompact cardinal for proper posets implies SDLS$^-$($\mathcal{L}^{\aleph_0}_{stat}$,$<2^{\aleph_0}$), and the existence of a Laver generically supercompact cardinal for ccc posets implies SDLS $^{int}_+$($\mathcal{L}^{PKL}_{stat}$,$<2^{\aleph_0}$).

In the talk I shall explain the details of these principles and discuss about known results around them.

[1] S.F., André Ottenbreit Maschio Rodrigues and Hiroshi Sakai, Strong downward Löwenheim-Skolem theorems for stationary logics, II — reflection down to the continuum, submitted.
`https://fuchino.ddo.jp/papers/SDLS-III-x.pdf`

[2] Bernhard König, Generic compactness reformulated, Archive of Mathematical Logic 43, (2004), 311 – 326.